# DIGITAL COMPUTER PROGRAM FOR FLEXURAL ANALYSIS OF BEAMS

GARY B. LOWE

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by

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Submitted in partial fulfillment of
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## Abstract.

This paper describes a solution to the flexural analysis of a beam by digital computer. Subroutines were written which calculate shear, moment, slope and deflection at any specified point on a beam if the loading is known. Shear deflection may be included for many support conditions. Neither elastic supports nor column effects are considered. The subroutines solve problems of variable beam cross-section and loading by utilizing an extension of "McCaulay's Method", a generalized step function.

The main program provides input-output facilities and also solves for indeterminate reactions on the beam.

Problems and their solutions, showing the versatility of this program, are also included.

## Acknowledgment.

The author wishes to thank Professor Gilles Cantin for his continued advice, direction, and encouragement. The director and staff of the computer center were very generous with their help and time. Professor John E. Brock's assistance was also appreciated.

## List of Illustrations

## 1. Introduction.

The routine solution to engineering problems by digital computer appears to be limited only by the scope of the available programs. A flexible program permitting the solution of routine problems in flexural analysis of beams would seemingly fill a need in the field of digital computer applioations to Mechanics. The present work is devoted to suoh an idea.

Five subroutines, "LOAD", "SHEAR", "MOMENT", "SLOPE", and "DEFLECT", were written which will calculate the load, shear, moment, slope, and deflection, respectively, at any indicated point on a beam if the indeterminate quantities are calculated elsewhere. The use of generalized step functions in this project allows for piecewise linear variation of distributed loading, bending compliance, $\frac{1}{EI}$, and shear compliance, $\frac{K}{AG}$.

The main program, "BEAM3", first generates and then solves a set of simultaneous linear equations, calculating the indeterminate quantities of the beam. The above mentioned subroutines are then utilized to calculate the remaining flexural quantities. This program also provides input-output facility.

## 2. Definition of the problem.

The problem of structural beam analysis is one in which the external loading applied to the beam is well defined, as is the geometric configuration and the nature of the beam itself. Where information about such a beam is required, this information will most likely fall into the following categories:

1. Internal shear forces, V, and moments, M.
2. Slope, $\theta$.
3. Deflection, y.
4. Redundant reactions, either forces, Ry, or moments, $RM_z$.

"Flexural analysis" is used here as meaning the process by which this information is generated from the known data.

In attacking this problem of flexural analysis, the intention was to limit the scope as little as possible. However, several customary simplifications and limitations were made. First, elastic supports and beam column effects were not considered. Limitations concerning the type and number of external loads and the geometry of the beam will be discussed later. Also the basic relations which follow, inherently restrict the deflection of the beam to small values and the internal stresses to values below the elastic limit of the structural material. Shear deflection, which is generally neglected in a problem of this type, has been included in some problems.

### a. Bending deflection ($y_1$).

The Bernoulli-Euler-Navier equation governing the elastic curve of a beam due to bending, consistent with the sign convention shown in Appendix B is:

$$\frac{M}{EI} = \text{curvature} = (d^2y/dx^2)\left[1+(dy/dx)^2\right]^{-3/2}$$

and we use the approximation, curvature $\approx d^2y/dx^2$.

The following equations, derived from the above equation and shown in Timoshenko /5/,[1] with differences due to the chosen sign convention, describe the flexure of a beam (See Appendix B for notation).

$$W = q(x) \quad ,$$

$$V = \int W dx \quad ,$$

$$M = \int V dx \quad ,$$

$$\theta_1 = \frac{dy_1}{dx} = \int \frac{M}{EI} dx \quad ,$$

$$y_1 = \int \theta_1 \, dx \quad ,$$

b. Shear deflection ($y_2$).

The equation governing the shear deflection of a beam, also given in Timoshenko /5/, compatible with the stated sign convention becomes:

$$\frac{dy_2}{dx} = -\frac{KV}{AG}$$

It is evident that:

$$y_2 = \int \left(-\frac{KV}{AG}\right) dx$$

c. Total deflection (y).

The equations used in this thesis are the result of the sum of the above shear and bending deflections. They are:

$$W = q(x) \quad ,$$

[1] Numbers so indicated refer to the bibliography.

$$V = \int_0^x W dx + \Sigma F_1{}^{*}$$

$$M = \int_0^x V dx + \Sigma M_1{}^{*}$$

$$\theta = \int_0^x \frac{M}{EI} dx - \frac{KV}{AG} + C_1{}^{**}$$

$$Y = \int_0^x \theta \, dx + C_2{}^{**}$$

Note that step-discontinuities may appear in all of the above equations, except in the equation for deflection. These discontinuities limit the methods that can be used in handling the equations.

Three methods that could be used to solve the equations are a pure analytical approach, a step-by-step numerical integration scheme, and the "curly bracket" method. Of these methods, the analytical approach is generally restricted to a specific problem and is unsuited for adaptation to digital computer programming. The numerical integration scheme can be used in the computer solution of flexural analysis and may have been programmed; however, our search of literature has not uncovered any such program. The "curly bracket" method was chosen for use in this thesis because it was felt that it would result in a more exact solution requiring less

*In these equations, $\Sigma F_1$ and $\Sigma M_1$, represent the sum of the point forces and moments, respectively, applied to the beam.

**$C_1$ and $C_2$ are constants of integration.

computer time, particularly if results were required at only a few points.

3. "Curly bracket" method

The "curly bracket" method is based upon the properties of a convenient notation defined as follows:

$$\{x-a\}^n = 0 \qquad \text{if } x<a$$
$$= (x-a)^n \qquad \text{if } x>a$$

where n is an integer$\geqq 0$.

Appendix A contains further properties of expressions containing curly brackets. The notation is generally credited to McCaulay /1/, but similar, though less useful notations can be traced to A.Föppl and St. Venant. Also, the ideas involved are related to the use of the Laplace Transformation with the result that this method or approximations to it are reinvented frequently /8/ /9/.

Using this method allows one equation, valid over the entire length of the beam, to be expressed for each of the basic equations of flexure. The particular value of this method is that it permits the integration of these equations to be carried out easily and has the virtue of adjusting the constants of integration most conveniently.

In addition, for use with a digital computer, the "switching" property of the curly bracket can be easily handled in FORTRAN by use of an "IF" statement.

A simple example should convince most readers of the convenience of this method. Examine the following problem, which consists of a uniform beam, simply supported at each end with a concentrated load at the center of the span.

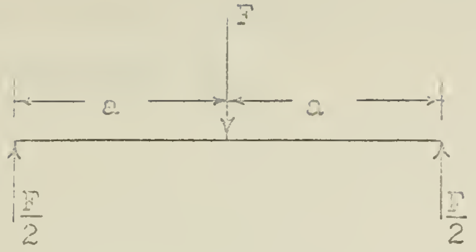


Fig. 1

The following equations lead to the "solution" of this problem.

$$V = \frac{F}{2} - F\{x-a\}^{0}$$

$$M = \frac{F}{2}x - F\{x-a\}^{1}$$

$$EI\theta = F\frac{x^2}{4} - F\frac{\{x-a\}^{2}}{2} + C_1$$

$$EIy = F\frac{x^3}{12} - F\frac{\{x-a\}^{3}}{6} + C_1x + \cancel{C_2}^{\,0}$$

The only unknown value, $C_1$, in the equations can easily be calculated, using the boundary condition, y=0 at x=2a.

$$C_1 = \frac{-8Fa^2}{24} + \frac{Fa^2}{12} = -\frac{Fa^2}{4}$$

Thus the resulting equation defining deflection at any point along the beam is:

$$y = \frac{1}{EI}\left[F\frac{x^3}{12} - F\frac{\{x-a\}^{3}}{6} - F\frac{xa^2}{4}\right]$$

As McCaulay used this method, it was suitable only for beams of uniform cross-section. Quinlan /3/ suggested a way of expanding the method to include beams with piecewise con-

stant variation of bending compliance, $\frac{1}{EI}$. In this thesis the method has been further expanded to include beams with piecewise linear variation of both bending compliance and shear compliance, $\frac{K}{AG}$.

## 4. Implementation.

Although the formulas shown in Appendix A could be expanded further, a piecewise linear variation of distributed loads, bending compliance, and shear compliance was considered capable of providing sufficient accuracy and flexibility for the resulting program.

Fig. 2(a) illustrates a distributed load on a beam whose origin is at x=0. Fig. 2(b) shows an element of the distributed load.

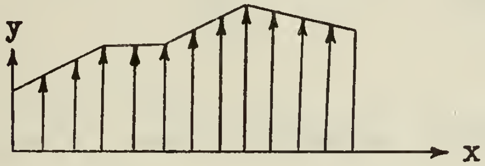


Fig. 2(a)

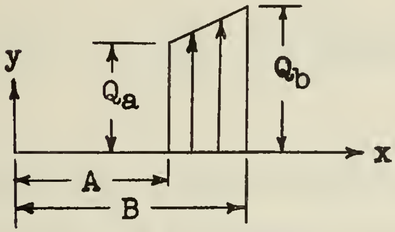


Fig. 2(b)

The analytical expression for this element is:

$$Q(x) = Q_a\{x-A\}^0 + \frac{Q_b-Q_a}{B-A}\{x-A\}^1 - Q_b\{x-B\}^0 - \frac{Q_b-Q_a}{B-A}\{x-B\}^1$$

The same relationship is used for an element, f(x), of bending compliance and an element, g(x), of shear compliance upon substituting f for Q and g for Q respectively.

The basic equations of flexure can now be expressed in the same manner as they were utilized in programming. They

are:[2]

$$W(x) = \sum_{i=1}^{NOL} Q_i(x) \tag{1}$$

$$V(x) = \int_0^x W(x)dx + \sum_{i=1}^{NOF} F_{y_i}\{x-x_{f_i}\}^0 + \sum_{i=1}^{NOR} R_{y_i}\{x-x_{r_i}\}^0 \tag{2}$$

where $F_{y_i}$ are upward, known forces at $x=x_{f_i}$ and $R_{y_i}$ are upward, unknown reactions at $x=x_{r_i}$

$$M(x) = \int_0^x V(x)dx + \sum_{i=1}^{NOM} M_{z_i}\{x-x_{m_i}\}^0 + \sum_{i=1}^{NORM} RM_{z_i}\{x-x_{rm_i}\}^0 \tag{3}$$

where $M_{z_i}$ are clockwise, known moments at $x=x_{m_i}$ and $RM_{z_i}$ are clockwise, unknown reactive moments at $x=x_{rm_i}$

$$\theta = \frac{dy}{dx} = \int_0^x \left[M(x) \sum_{i=1}^{NOI} f_i(x)\right]dx - \left[V(x) \sum_{i=1}^{NOK} g_i(x)\right] + C_1 \tag{4}$$

where $C_1$ is the constant of integration.

Appendix A indicates how the product and the integration of the product of the curly bracket symbols are formed.

$$y = \int_0^x\int_0^x \left[M(\xi)\sum_{i=1}^{NOI} f_i(\xi)\right] d\xi \, dx - \int_c^x \left[V(x)\sum_{i=1}^{NOK} g_i(x)\right]dx + C_1x + C_2 \tag{5}$$

where $C_2$ is the constant of integration.

Equations (1) through (5) when properly coded in FORTRAN

[2] The numbers, NOL, NOF, NOR, NORM, NOI and NOK, appearing above the summation signs are program input parameters indicating the number of terms for each problem.

form the core of the five basic subroutines: LOAD, SHEAR MOMENT, SLOPE and DEFLECT. It must be noted before discussing the subroutines that equations (2) through (5) involve the unknown values $C_1$, $C_2$, $R_{y_i}$, and $RM_{z_i}$, where there may be as many as ten values each of $R_y$ and $RM_z$. The calculation of these values is done in the main program BEAM3 and will be discussed in section 6.

5. The basic subroutines.

Subroutines LOAD, SHEAR and MOMENT.

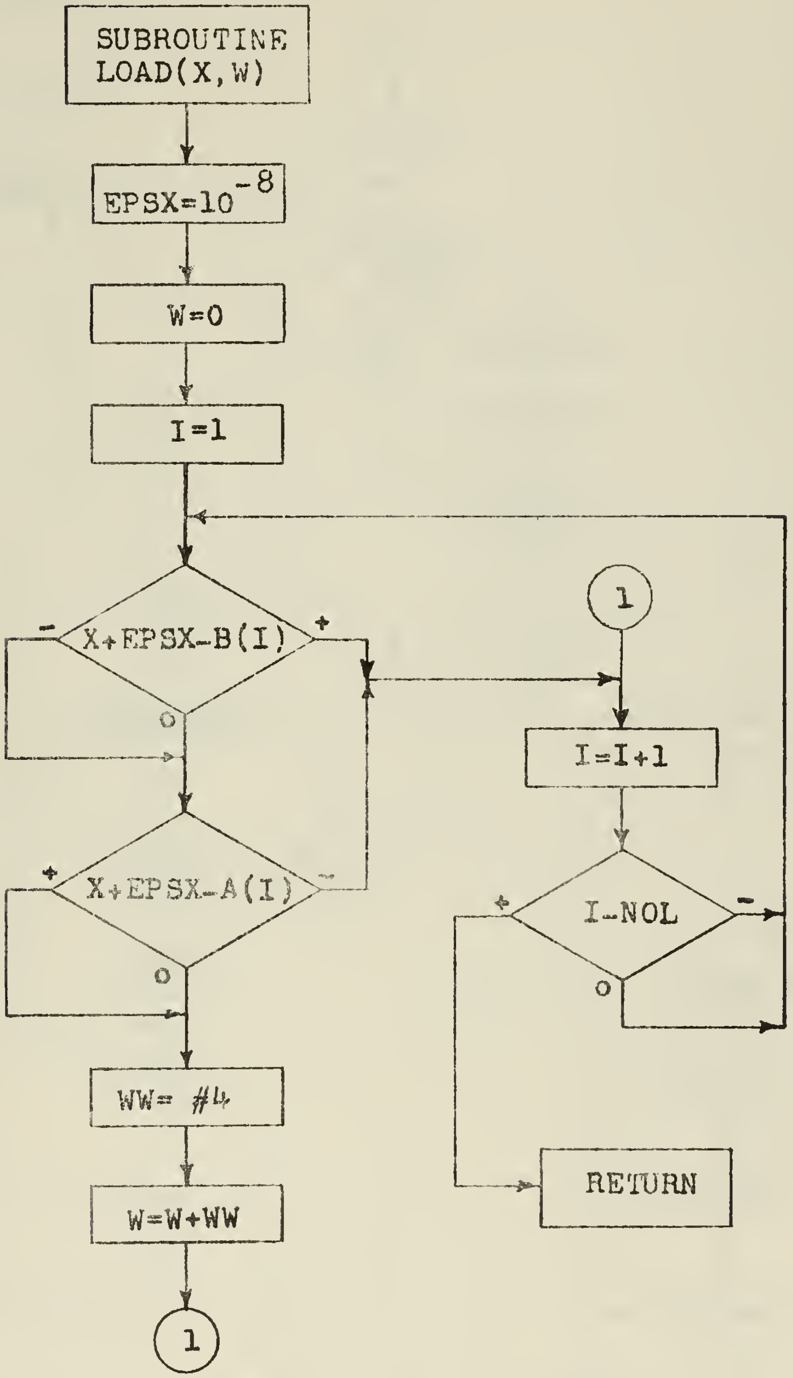
The above named subroutines, as their names imply, are employed to calculate the total distributed load, shear and moment, respectively, at any desired point on the beam. The expanded forms of equations (1), (2) and (3) were used in writing these subroutines. Utilizing the distance, $x_i$, as the entering argument, the subroutines select and sum the appropriate terms in the corresponding equation by inspecting all loading terms and discarding those in which the value within the curly bracket is negative. Actually the load, shear and moment are calculated a small increment, $\mathcal{E} = 10^{-8}$, to the right of $x_i$ in order to obtain the right side value of any step-discontinuities occuring at the point, $x_i$ (See flow charts on pages 13, 14 and 15).

Subroutine SLOPE.

This subroutine is equation (4), excluding the constant of integration, $C_1$, in FORTRAN language (See flow chart on page 17). Note that the equation can be expressed as a summation of terms, which are constants times the multiplication of two step-functions of the form $C\{x-a\}^n\{x-b\}^m$, or the summation of the integral of the same type of terms, where n equals either 0 or 1.

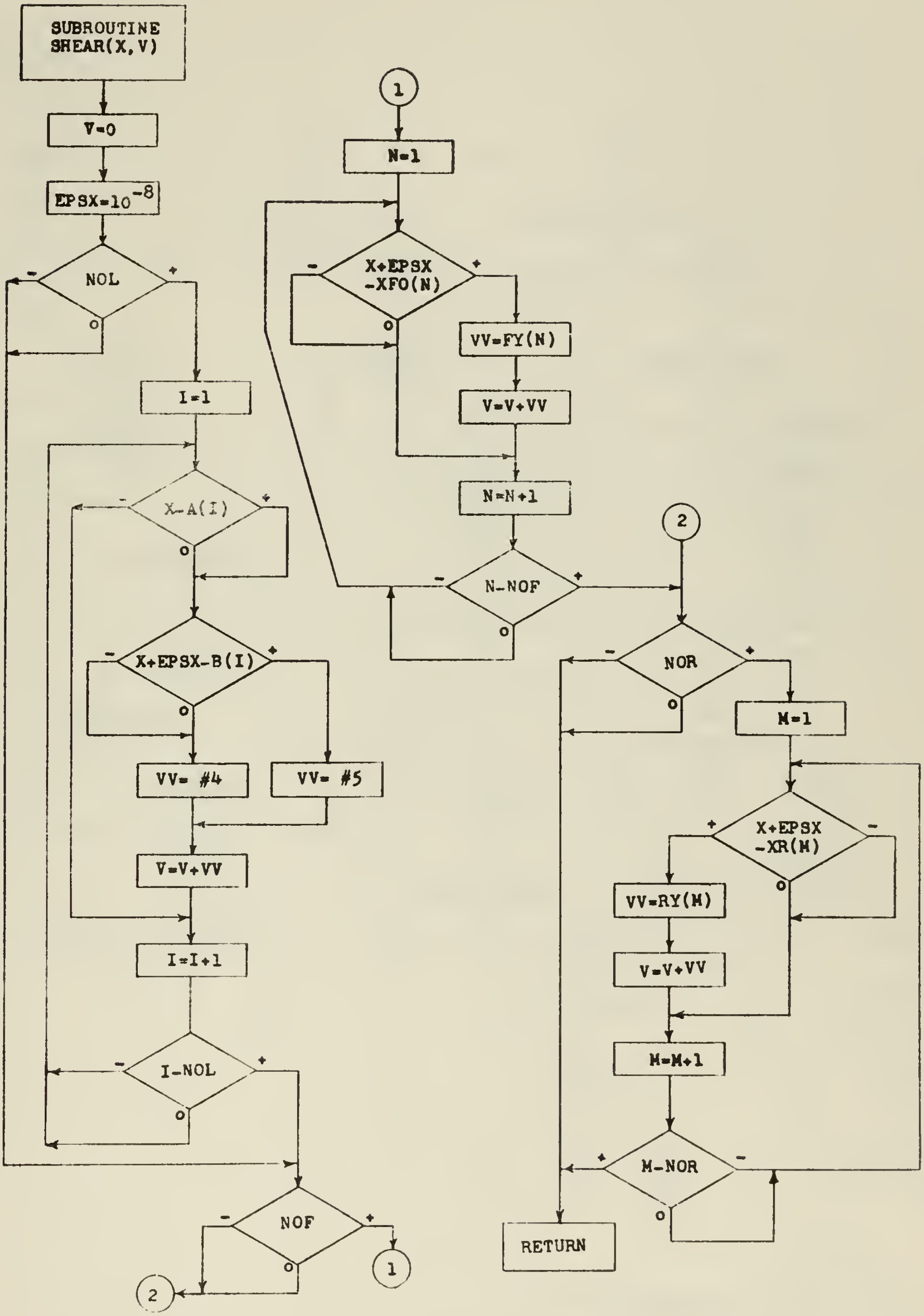
Function subroutines, FTON and FT1N (see Appendix A), will calculate the value of $\{x-a\}^o\{x-b\}^m$ and $\{x-a\}^1\{x-b\}^n$, respectively. Function subroutines, ENTON and ENT1N (See

Flow Chart of LOAD



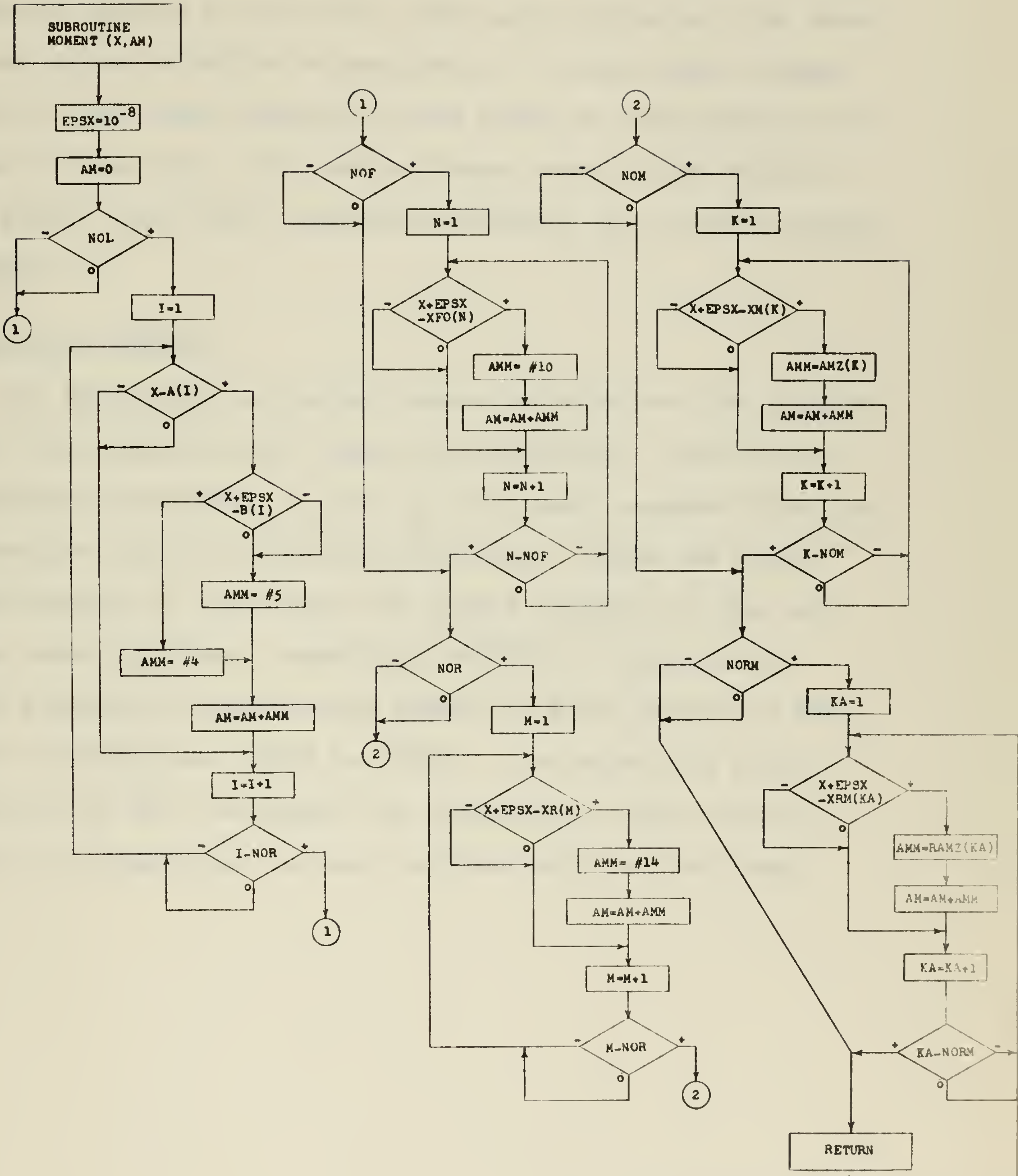
Note: Numbers preceded by a number sign, #, refer to the statement number in the listing of LOAD.

Flow Chart of SHEAR



Note: Numbers preceded by a number sign, #, refer to the statement number in the listing of SHEAR.

Flow Chart of MOMENT



Note: Numbers preceded by a number sign, #, refer to the statement number in the listing of MOMENT.

Appendix A), will calculate the first integral of these functions. Equation (4) can now be expressed as the summation of constants multiplied by the appropriate function subroutines.

SLOPE calculates all combinations of M(x) and f(x) by stepping through all the f(x) terms and varying all the terms in the moment equation at each step. It then steps through all the g(x) terms varying all the terms in the equation for shear at each step. The sum of these terms is the value of the slope, minus the integration constant, $C_1$, at the entered distance, x.

Subroutine DEFLECT.

At this point the author wishes to point out the similarity of the equations for slope and deflection. Since the integration constants, $C_1$ and $C_2$, are again excluded from the subroutine, and the function subroutines, DITON and DIT1N (See Appendix A), calculate the second integral of the multiple step-functions, subroutine DEFLECT is identical to SLOPE except for substituting ENTON for FTON, ENT1N for FT1N, DITON for ENTON and DIT1N for ENT1N. The result is a subroutine that will calculate the deflection, minus the constants of integration, at any required point on the beam.

6. Program BEAM3.

The main program, BEAM3, can be divided into four distinct steps. These steps are: reading the input data, calculation of indeterminate quantities, calculation of the flexural quantities, and the output of the solution.

The program first reads the input data which includes the length of the beam, all external loading, the location of all reactions, the flexural properties of the beam, and a group of parameters giving the number of each type of load or reaction. It also provides for reading a predetermined value of deflection, PDEFT, at each reaction and a predetermined value of slope, PSLP, at each moment reaction.

The program then proceeds to solve for the indeterminate quantities of the beam by utilizing the two equations of static equilibrium plus equations obtained from the known boundary condition at each reaction or reactive moment. The first two equations are simply the summation of the forces on the beam equals zero and the summation of the moments about the right end of the beam equals zero. The remainder of the equations are obtained from the fact that the slope at each reactive moment or the deflection at each reaction is known. Using this fact in the appropriate one of either equation (4) or (5) results in a set of linear equations for the indeterminate quantities.

The equations thus obtained are generated in a matrix form that is compatible with subroutine GAUSS2 (See Appendix E). This subroutine is then called to solve the equations simulta-

neously. If the matrix is singular, GAUSS2 exits to the calling program which in turn prints "MATRIX SINGULAR" and then stops. If not, the subroutine solves the equations and returns the calculated values to the main program. The most serious limitations of this program are imposed by the generation or solution of the simultaneous equations, therefore it is appropriate to include and explain several of them at this point.

First, at least one reaction, even if it is later forced to equal zero, must be included with the input, or a singular matrix will be generated.

Including shear deflection in the solution must also be greatly limited at this point. Since in general the shear includes step discontinuities, they will also appear in the slope if shear deflection is included. The program generates an equation for slope at each moment reaction. If by chance, either a reaction or point force occurs at the same point as a moment reaction, the slope at this point is double valued. The program is unable to distinguish which of the two values is required except when it occurs at the extreme ends of the beam. For example a beam "built in" at both ends is acceptable. If the beam is "built in" at only one end, it must be solved in a manner such that the "built in" is at the origin of the beam.

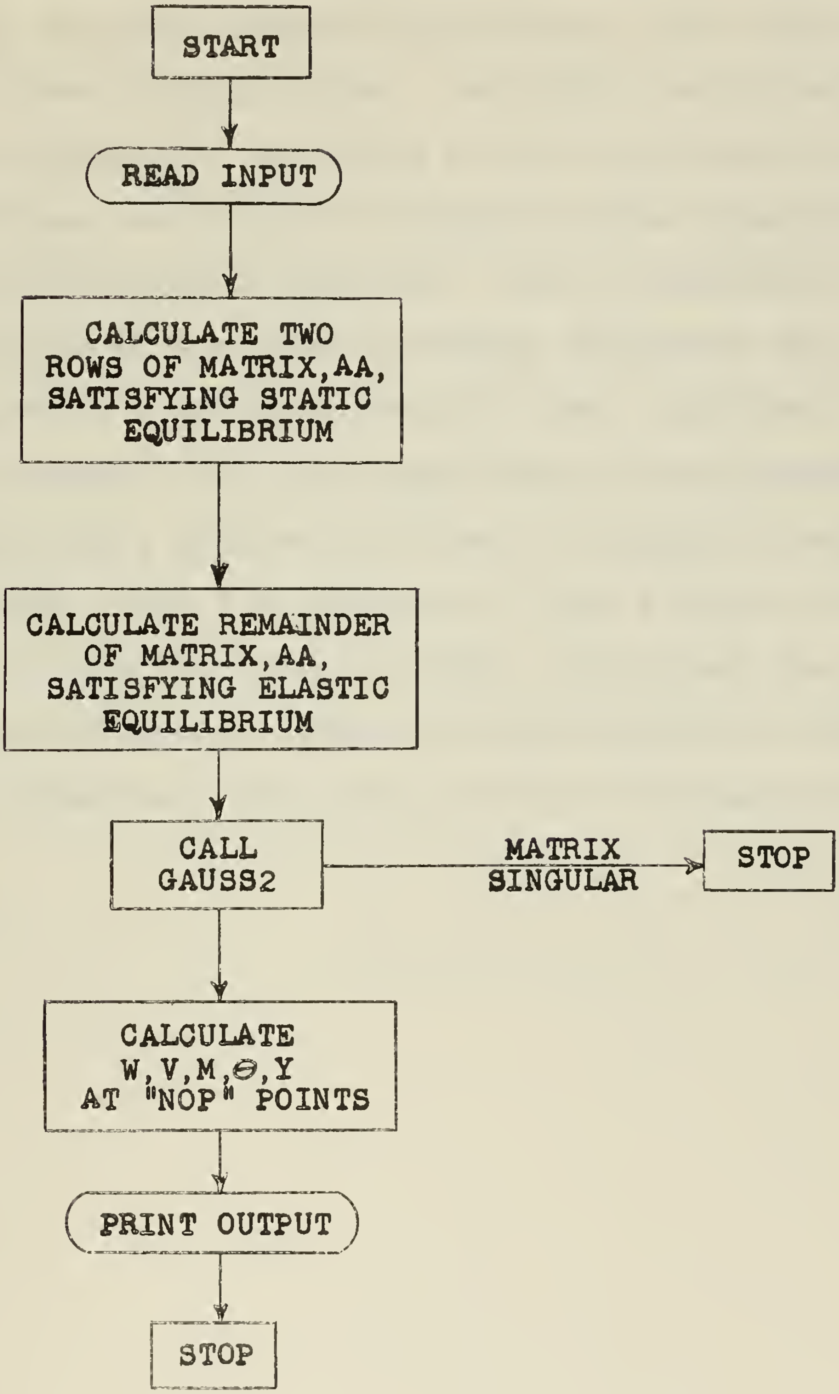
The program then divides the beam into a number of increments, NOP-1,* and calls the subroutines, LOAD, SHEAR,

*NOP is another parameter read in at the start of BEAM3.

MOMENT, SLOPE and DEFLECT to calculate the flexural quantities at each of these increments.

The output consists of printing the distance from the left end of the beam to the point in question and the values of the load, shear, moment, slope and deflection at that point. The values of all of the indeterminate quantities are also printed out.

Fig. 7 Simplified Flow Chart of BEAM3


START
READ INPUT
CALCULATE TWO ROWS OF MATRIX, AA, SATISFYING STATIC EQUILIBRIUM
CALCULATE REMAINDER OF MATRIX, AA, SATISFYING ELASTIC EQUILIBRIUM
CALL GAUSS2
MATRIX SINGULAR
STOP
CALCULATE W, V, M, Θ, Y AT "NOP" POINTS
PRINT OUTPUT
STOP

[illegible]

## 7. Testing of Program.

The testing of the program and the subroutines could, in theory, continue indefinitely because of the unlimited number of beam configurations. The author has attempted to show (See Appendix D) solutions to test problems of simple configurations and constant bending and shear compliance to compare with classical solutions. Then a statically determinate problem with a varying bending compliance was solved and compared to a solution obtained from a numerical integration scheme. The most rigid test for the program was in constructing a problem for a beam of unlikely configuration (See Problems 6 and 7 of Appendix D), and solving the flexural analysis of this beam from both ends. Note that the only significant difference between the two solutions is the opposite signs for shear and slope, which would of course, be expected.

## 8. Conclusions and Recommendations.

The thesis shows a method of including piecewise linear variation of both bending compliance and shear compliance in the flexural analysis of beams by extending "McCaulay's Method". Adapting this idea to the digital computer resulted in a program capable of analyzing a very general type of beam. However, had time permitted, several changes would have been made to the program. First, internal monitoring of the input data would be an asset to any user. Also, using the same general method of attack, this program would have been expanded to include bending compliance and shear compliance of a piecewise parabolic nature.

Other additions to the program, much larger in scope, would be including elastic supports and/or beam column effect.

It would be interesting to compare the results of this program to those obtained from a program utilizing the numerical integration method. Also, a more exhaustive study of the program could be made to determine accuracy of, and time requirement for, solutions.

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# APPENDIX A

## "Curly bracket" method, definitions and formulas

We define:

1. $$\{x-a\}^n = (x-a)^n \quad , \quad x > a$$
$$= 0 \quad , \quad x < a$$

where n is an integer $\geqq 0$

From this definition, it is possible to establish the following formulas. Although it would be possible to express these results in terms of curly brackets, in most cases the resulting expressions would be rather complicated. The forms exhibited below have the advantage of being most readily conformable to the decision commands available in FORTRAN.

2. $$\int_0^x \{x-a\}^n dx = \frac{(x-a)^{n+1}}{n+1} \quad , \quad x > a$$
$$= 0 \quad , \quad \text{Otherwise}$$

3. $$\{x-a\}^0 \{x-b\}^n = (x-b)^n \quad , \quad x > a \text{ and } x > b$$
$$= 0 \quad , \quad \text{Otherwise}$$

4. $$\{x-a\}^1 \{x-b\}^n = (x-b)^{n+1} + (b-a)(x-b)^n \quad , \quad x > a \text{ and } x > b$$
$$= 0 \quad , \quad \text{Otherwise}$$

5. $$\int_0^x \{x-a\}^0 \{x-b\}^n dx = \frac{(x-b)^{n+1}}{n+1} - \frac{(a-b)^{n+1}}{n+1} \quad , \quad x > a > b$$
$$= \frac{(x-b)^{n+1}}{n+1} \quad , \quad x > b \geq a$$
$$= 0 \quad , \quad \text{Otherwise}$$

6. $$\int_0^x \{x-a\}^1 \{x-b\}^n dx = \frac{(x-b)^{n+2}}{n+2} + \frac{(b-a)(x-b)^{n+1}}{n+1} + \frac{(a-b)^{n+2}}{(n+1)(n+2)} \quad , \quad x>a>b$$

$$= \frac{(x-b)^{n+2}}{n+2} + \frac{(b-a)(x-b)^{n+1}}{n+1} , \quad x>b>a$$

$$= 0 \quad , \text{ Otherwise}$$

7. $$\int_0^x \int_0^x \{\xi-a\}^0 \{\xi-b\}^n d\xi \, dx$$

$$= \frac{(x-b)^{n+2}}{(n+1)(n+2)} - \frac{(x-a)(a-b)^{n+1}}{(n+1)} - \frac{(a-b)^{n+2}}{(n+1)(n+2)} \quad , \quad x>a>b$$

$$= \frac{(x-b)^{n+2}}{(n+1)(n+2)} \quad , \quad x>b\geq a$$

$$= 0 \quad , \quad \text{Otherwise}$$

8. $$\int_0^x \int_0^x \{\xi-a\}^1 \{\xi-b\}^n d\xi \, dx$$

$$= \frac{(x-b)^{n+3}}{(n+2)(n+3)} + \frac{(b-a)(x-b)^{n+2}}{(n+1)(n+2)} + \frac{(x-a)(a-b)^{n+2}}{(n+1)(n+2)} + \frac{2(a-b)^{n+3}}{(n+1)(n+2)(n+3)} \quad , \quad x>a>b$$

$$= \frac{(x-b)^{n+3}}{(n+2)(n+3)} + \frac{(b-a)(x-b)^{n+2}}{(n+1)(n+2)} \quad , \quad x>b>a$$

$$= 0 \quad , \text{ Otherwise}$$

Function Subroutines FTON, FT1N, ENTON, ENT1N, DITON, DIT1N

The above named function subroutines were written and utilized to evaluate each of the preceeding formulas numbered 3 through 8. The values x, a, b and n were used as entering

arguments to these functions. The function then became the variable as follows:

$$\text{FTON} = \{x-a\}^{0}\{x-b\}^{n}$$

$$\text{FT1N} = \{x-a\}^{1}\{x-b\}^{n}$$

$$\text{ENTON} = \int_{0}^{x} \{x-a\}^{0}\{x-b\}^{n}dx$$

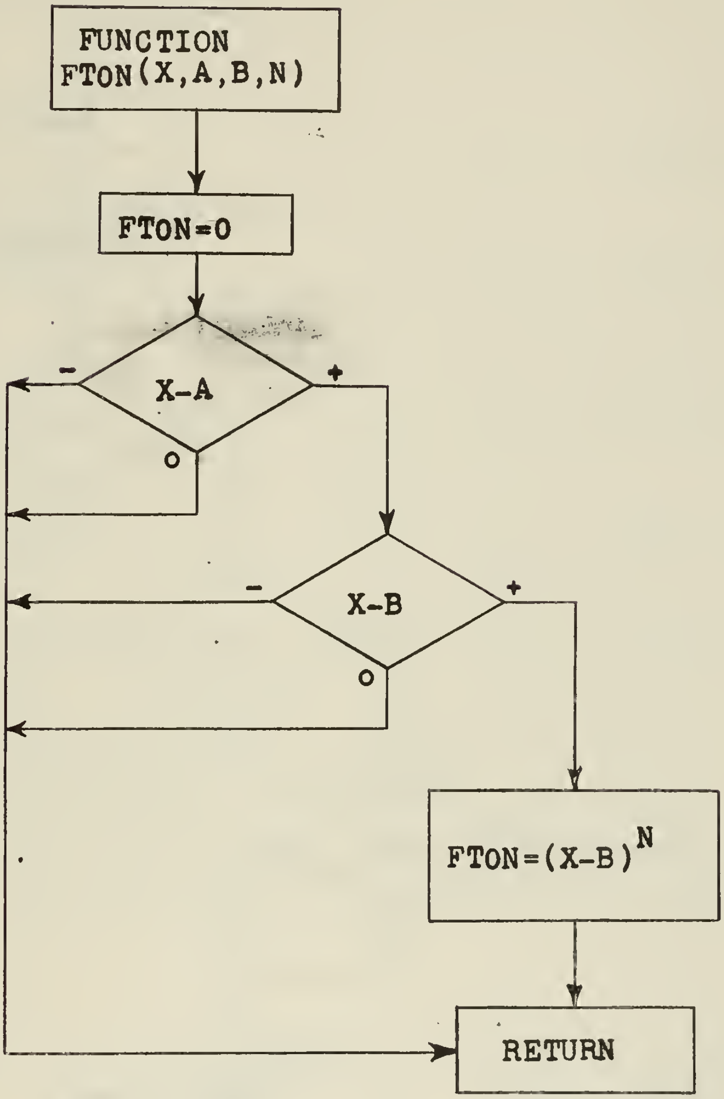
$$\text{ENT1N} = \int_{0}^{x} \{x-a\}^{1}\{x-b\}^{n}dx$$

$$\text{DITON} = \int_{0}^{x}\int_{0}^{x} \{\xi-a\}^{0}\{\xi-b\}^{n}d\xi\, dx$$

$$\text{DIT1N} = \int_{0}^{x}\int_{0}^{x} \{\xi-a\}^{1}\{\xi-b\}^{n}d\xi\, dx$$

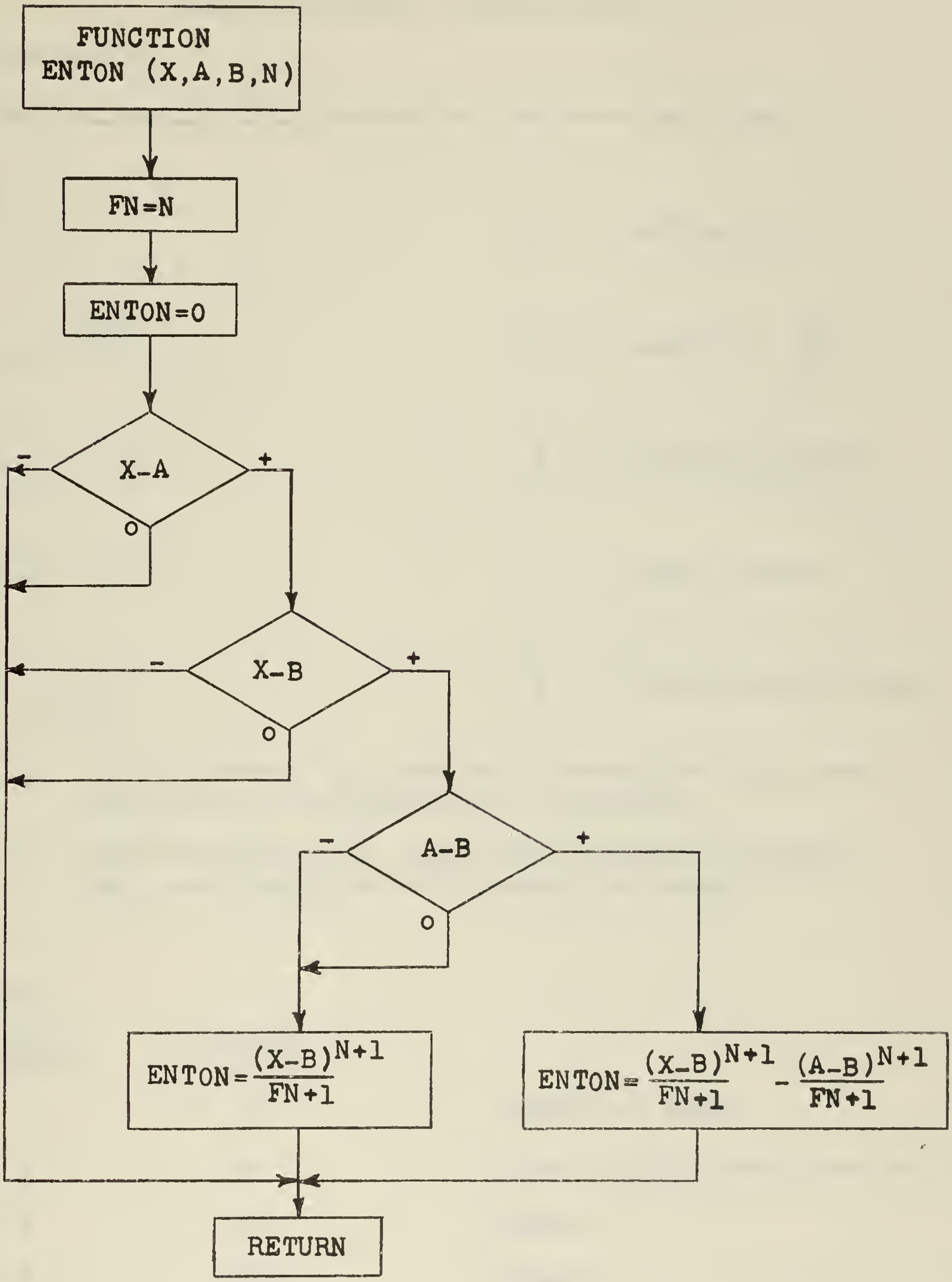
As examples, flow charts for FTON and ENTON are shown in Figs. 8 and 9.

Fig. 8 Flow Chart of FTON



[illegible]

Fig. 9   Flow Chart of ENTON


FUNCTION
ENTON (X,A,B,N)
FN=N
ENTON=0
X-A
-
+
0
X-B
-
+
0
A-B
-
+
0
$ENTON=\frac{(X-B)^{N+1}}{FN+1}$
$ENTON=\frac{(X-B)^{N+1}}{FN+1}-\frac{(A-B)^{N+1}}{FN+1}$
RETURN

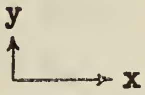
[illegible]

## APPENDIX B

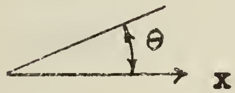
### Sign Convention and Notation

### Sign Convention

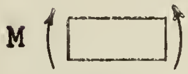
The following sign convention has been employed:



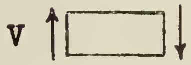
y = deflection



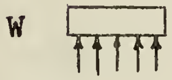
$\theta$ = slope = $\frac{dy}{dx}$



M = bending moment



V = shear force



W = distributed loads

Note: 1. Point forces, $F_y$, and point reactions, $R_y$, are positive in the positive y direction.

2. Point moments, $M_z$, and point reactive moments, $RM_2$, are positive in a clockwise sense.

### Notation:

| THESIS | PROGRAM | |
|---|---|---|
| | C | Length of beam |
| W | W | Total distributed load at a point |
| V | V | Shear |
| M | AM | Moment |
| $\theta$ | Y1 | Slope |
| y | Y | Deflection |

| | | |
|---|---|---|
| $F_y$ | FY | Point load or force |
| $R_y$ | RY | Point reaction |
| $M_z$ | AMZ | Point moment |
| $RM_z$ | RAMZ | Point reactive moment |
| $Q(x)$ | | Element of distributed load |
| $Q_a$ | AQ | Distributed load (left end) |
| $Q_b$ | QB | Distributed load (right end) |
| $f(x)$ | | Element of bending compliance $(\frac{1}{EI})$ |
| | EA | Bending compliance (left end of element) |
| | EB | Bending compliance (right end of element) |
| $g(x)$ | | Element of shear compliance $(\frac{K}{AG})$ |
| | GA | Shear compliance (left end of element) |
| | GB | Shear compliance (right end of element) |
| $X_f$ | XFO | Distance to $F_y$ |
| $X_r$ | XR | Distance to $R_y$ |
| $X_m$ | XM | Distance to $M_z$ |
| $X_m$ | RXM | Distance to $RM_z$ |
| $A_g$ | AG | Distance to $G_a$ |
| $B_g$ | BG | Distance to $G_b$ |
| $A_e$ | AE | Distance to $E_a$ |
| $B_e$ | BE | Distance to $E_b$ |
| A | A | Distance to $Q_a$ |
| B | B | Distance to $Q_b$ |
| NOL | NOL | Number of distributed loads (25)[3] |

[3] Numbers following explanation of terms NOL--NOK indicate the arbitarily chosen maximum value of each for this program.

|  |  |  |
|---|---|---|
| NOF | NOF | Number of point forces (100) |
| NOR | NOR | Number of point reactions (10) |
| NOM | NOM | Number of point moments (50) |
| NORM | NORM | Number of point reactive moments (10) |
| NOI | NOI | Number of bending compliance elements (25) |
| NOK | NOK | Number of shear compliance elements (25) |
| PDEFT | PDEFT | Predetermined deflection at a reaction |
| PSLP | PSLP | Predetermined slope at a reactive moment |
| NOP | NOP | Number of points at which the flexural quantities will be calculated. |

# APPENDIX C

## General Information Concerning Use of Program

The composite program, BEAM3 plus subroutines, was written in FORTRAN-60, compiled and tested on a Control Data Corporation 1604 digital computer. It consists of 412 FORTRAN statements requiring about 580 cards. The computer storage requirement is about 12,500 cells, however, this number can be reduced by changing the dimension statements which would of course, reduce either the maximum number of external loads, elements of shear compliance or bending compliance, or number of points at which flexural quantities could be calculated.

The FORTRAN statements used in this program are: GO TO, computed GO TO, IF, STOP, DO, CONTINUE, FORMAT, READ, PRINT, WRITE OUTPUT TAPE, FUNCTION, SUBROUTINE, RETURN, CALL, DIMENSION, COMMON, and END.

There are no conversion constants incorporated in this program. As such all input must be in a consistant set of units.

## "FORMAT" for Input Data

The input data for a problem is read by the program with a FORMAT of I4 for all "fixed point" variables and F20.0 for all floating point variables. The arrangement of the data on the input cards is shown as follows:

| Group no. | No. of cards in group | Fields | Entries |
|---|---|---|---|
| 1 | 1 | F20.0 | C |
| 2 | 1 | 8I4 | NOP, NOL, NOF, NOR<br>NOM, NORM, NOI, NOK |
| 3 | NOL | 4F20.0 | A, B, $A_a$, $A_b$ |
| 4 | NOF | 2F20.0 | $x_f$, $F_y$ |
| 5 | NOR | 3F20.0 | $x_r$, 0.0, PDEFT |
| 6 | NOM | 2F20.0 | $x_m$, $M_z$ |
| 7 | NORM | 3F20.0 | $x_m$, 0.0, PSLP |
| 8 | NOI | 4F20.0 | $A_e$, $B_e$, $E_a$, $E_b$ |
| 9 | NOK | 4F20.0 | $A_g$, $B_g$, $G_a$, $G_b$ |

As an example the input data cards for test problem #1 in Appendix D was as follows:

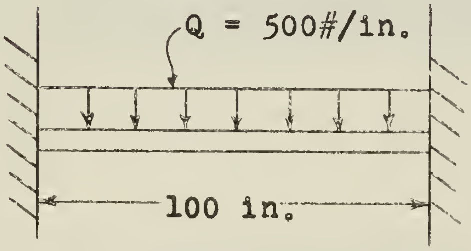
```
CARD 1:   100
CARD 2:   21  1  0  2  0  2  1  0
CARD 3:    0.        100        -500.  -500.
CARD 4:    0.         0.         .0
CARD 5:   100         0.         .0
CARD 6:    0.         0.         0.
CARD 7:   100         0.         0.
CARD 8:    0.        100.   .000000001  .000000001
```

# APPENDIX D

## Test problems and solutions

### Test problem 1

This problem consists of a beam of uniform cross-section, "built-in" at both ends, and loaded with a uniformly distributed weight of 500#/in. (See Fig. 10)



$$\frac{1}{EI} = 10^{-9} \frac{1}{\#\text{in.}^2}$$

$$\frac{K}{AG} = 0$$

Fig. 10

### Test problem 2

This problem is identical to problem 1 except that the slope at either end of the beam is forced to values other than zero.

$$\theta_{\text{left end}} = -.002$$

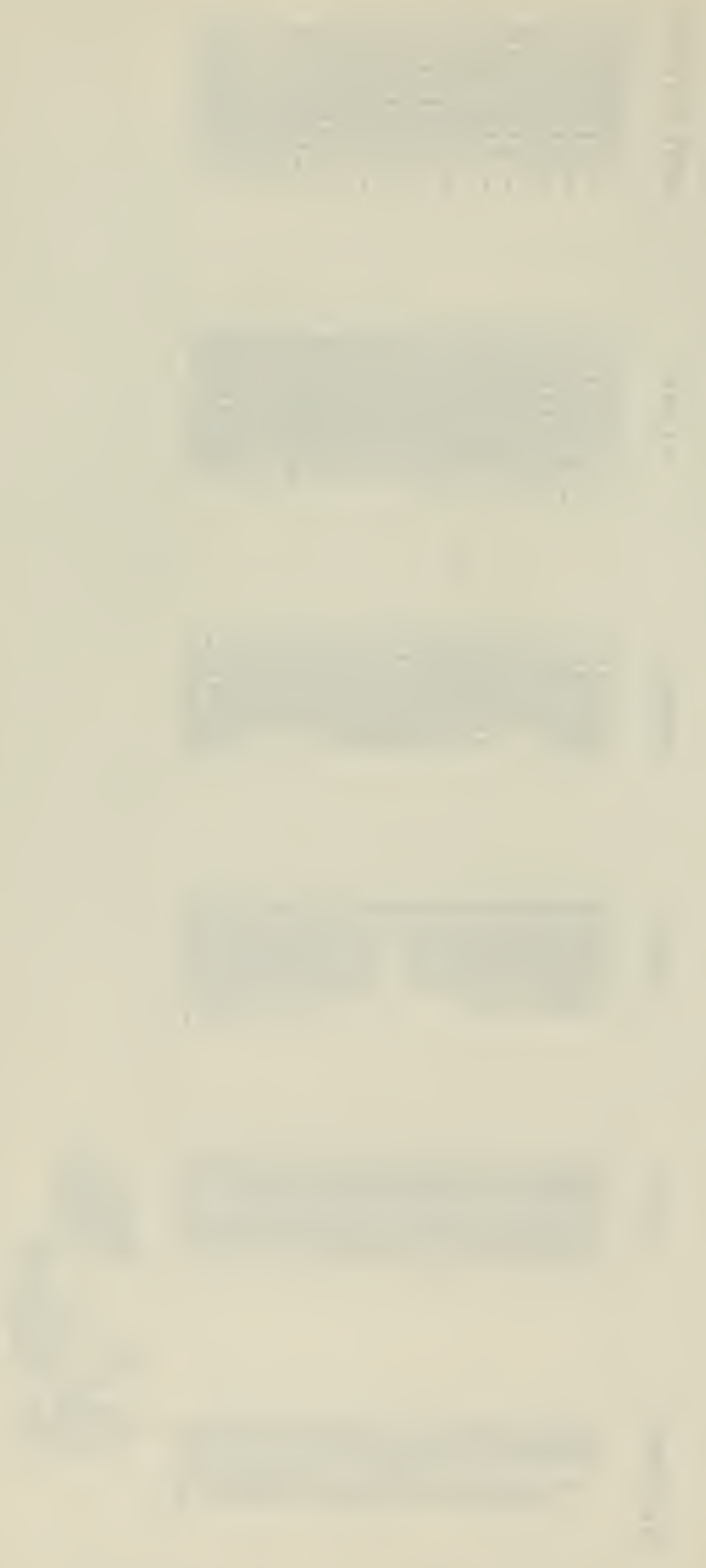
$$\theta_{\text{right end}} = .002$$

SOLUTION TO TEST PROBLEM 1

```
 DISTANCE        LOAD          SHEAR         MOMENT          SLOPE          DEFLECTION

     .00       -500.00       25000.0       -416667.      -.00000000       .00000000
    5.00       -500.00       22500.0       -297917.      -.00178125      -.00470052
   10.00       -500.00       20000.0       -191667.      -.00300000      -.01687500
   15.00       -500.00       17500.0        -97917.      -.00371875      -.03386719
   20.00       -500.00       15000.0        -16667.      -.00400000      -.05333333
   25.00       -500.00       12500.0         52083.      -.00390625      -.07324219
   30.00       -500.00       10000.0        108333.      -.00350000      -.09187500
   35.00       -500.00        7500.0        152083.      -.00284375      -.10782552
   40.00       -500.00        5000.0        183333.      -.00200000      -.12000000
   45.00       -500.00        2500.0        202083.      -.00103125      -.12761719
   50.00       -500.00          -.0         208333.       .00000000      -.13020833
   55.00       -500.00       -2500.0        202083.       .00103125      -.12761719
   60.00       -500.00       -5000.0        183333.       .00200000      -.12000000
   65.00       -500.00       -7500.0        152083.       .00284375      -.10782552
   70.00       -500.00      -10000.0        108333.       .00350000      -.09187500
   75.00       -500.00      -12500.0         52083.       .00390625      -.07324219
   80.00       -500.00      -15000.0        -16667.       .00400000      -.05333333
   85.00       -500.00      -17500.0        -97917.       .00371875      -.03386719
   90.00       -500.00      -20000.0       -191667.       .00300000      -.01687500
   95.00       -500.00      -22500.0       -297917.       .00178125      -.00470052
  100.00       -500.00      -25000.0       -416667.       .00000000       .00000000

        RY( 1)=      25C00.
        RY( 2)=      25C00.
        RAMZ( 1)=  -416667.
        RAMZ( 2)=   416667.
        C1= .00000C0C
        C2= .00000C00
```

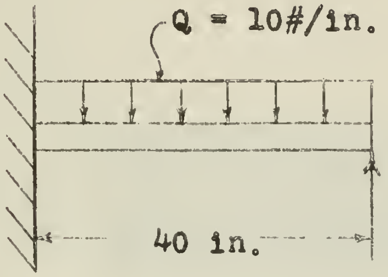
SOLUTION TO TEST PROBLEM 2

```
DISTANCE        LOAD          SHEAR          MOMENT          SLOPE          DEFLECTION

    .00       -500.00        25000.0        -376667.      -.00200000       .00000000
   5.00       -500.00        22500.0        -257917.      -.00358125      -.01420052
  10.00       -500.00        20000.0        -151667.      -.00460000      -.03487500
  15.00       -500.00        17500.0         -57917.      -.00511875      -.05936719
  20.00       -500.00        15000.0          23333.      -.00520000      -.08533333
  25.00       -500.00        12500.0          92083.      -.00490625      -.11074219
  30.00       -500.00        10000.0         148333.      -.00430000      -.13387500
  35.00       -500.00         7500.0         192083.      -.00344375      -.15332552
  40.00       -500.00         5000.0         223333.      -.00240000      -.16800000
  45.00       -500.00         2500.0         242083.      -.00123125      -.17711719
  50.00       -500.00           -.0          248333.       .00000000      -.18020833
  55.00       -500.00        -2500.0         242083.       .00123125      -.17711719
  60.00       -500.00        -5000.0         223333.       .00240000      -.16800000
  65.00       -500.00        -7500.0         192083.       .00344375      -.15332552
  70.00       -500.00       -10000.0         148333.       .00430000      -.13387500
  75.00       -500.00       -12500.0          92083.       .00490625      -.11074219
  80.00       -500.00       -15000.0          23333.       .00520000      -.08533333
  85.00       -500.00       -17500.0         -57917.       .00511875      -.05936719
  90.00       -500.00       -20000.0        -151667.       .00460000      -.03487500
  95.00       -500.00       -22500.0        -257917.       .00358125      -.01420052
 100.00       -500.00       -25000.0        -376667.       .00200000       .00000000

      RY( 1)=     25000.
      RY( 2)=     25000.
      RAMZ( 1)=  -376667.
      RAMZ( 2)=   376667.
      C1=-.00200000
      C2= .00000000
```



Test problem 3

This problem consists of a beam of uniform cross-section, "built-in" at one end and simply supported at the other, and loaded with a uniformly distributed weight. (See Fig. 11)



$$\frac{1}{EI} = .1332x10^{-7} \frac{1}{\#in.^2}$$

$$\frac{K}{AG} = 0$$

Fig. 11

Test problem 4

This problem is identical to problem number 3 except shear deflection is included in the solution.

$$\frac{K}{AG} = .159 \times 10^{-6} \ 1/\#in.$$

SOLUTION TO TEST PROBLEM 3

```
DISTANCE        LOAD        SHEAR       MOMENT        SLOPE        DEFLECTION

    .00       -10.00       250.00     -2000.00    -.00000000      .00000000
   2.00       -10.00       230.00     -1520.00    -.00004680    -.00004893
   4.00       -10.00       210.00     -1080.00    -.00008134    -.00017902
   6.00       -10.00       190.00      -680.00    -.00010470    -.00036683
   8.00       -10.00       170.00      -320.00    -.00011793    -.00059105
  10.00       -10.00       150.00          .00    -.00012210    -.00083250
  12.00       -10.00       130.00       280.00    -.00011828    -.00107412
  14.00       -10.00       110.00       520.00    -.00010754    -.00130101
  16.00       -10.00        90.00       720.00    -.00009093    -.00150036
  18.00       -10.00        70.00       880.00    -.00006953    -.00166154
  20.00       -10.00        50.00      1000.00    -.00004440    -.00177600
  22.00       -10.00        30.00      1080.00    -.00001661    -.00183736
  24.00       -10.00        10.00      1120.00     .00001279    -.00184136
  26.00       -10.00       -10.00      1120.00     .00004271    -.00178586
  28.00       -10.00       -30.00      1080.00     .00007211    -.00167086
  30.00       -10.00       -50.00      1000.00     .00009990    -.00149850
  32.00       -10.00       -70.00       880.00     .00012503    -.00127304
  34.00       -10.00       -90.00       720.00     .00014643    -.00100086
  36.00       -10.00      -110.00       520.00     .00016304    -.00069051
  38.00       -10.00      -130.00       280.00     .00017378    -.00035262
  40.00       -10.00      -150.00          .00     .00017760    -.00000000

     RY( 1)=     250.00
     RY( 2)=     150.00
     RAMZ( 1)=  -2000.00
     C1= .00000000
     C2= .00000000
```

SOLUTION TO TEST PROBLEM 4

```
DISTANCE        LOAD           SHEAR          MOMENT          SLOPE           DEFLECTION

    .00        -10.00         248.91        -1956.22       -.00003958         .00000000
   2.00        -10.00         228.91        -1478.41       -.00008206       -.00012375
   4.00        -10.00         208.91        -1040.59       -.00011234       -.00032009
   6.00        -10.00         188.91         -642.78       -.00013149       -.00056570
   8.00        -10.00         168.91         -284.97       -.00014058       -.00083936
  10.00        -10.00         148.91           32.84       -.00014067       -.00112203
  12.00        -10.00         128.91          310.65       -.00013283       -.00139677
  14.00        -10.00         108.91          548.46       -.00011812       -.00164877
  16.00        -10.00          88.91          746.27       -.00009760       -.00186536
  18.00        -10.00          68.91          904.08       -.00007235       -.00203602
  20.00        -10.00          48.91         1021.89       -.00004343       -.00215232
  22.00        -10.00          28.91         1099.70       -.00001190       -.00220799
  24.00        -10.00           8.91         1137.51        .00002117       -.00219889
  26.00        -10.00         -11.09         1135.32        .00005471       -.00212300
  28.00        -10.00         -31.09         1093.14        .00008766       -.00198044
  30.00        -10.00         -51.09         1010.95        .00011896       -.00177345
  32.00        -10.00         -71.09          888.76        .00014753       -.00150642
  34.00        -10.00         -91.09          726.57        .00017232       -.00118585
  36.00        -10.00        -111.09          524.38        .00019225       -.00082039
  38.00        -10.00        -131.09          282.19        .00020626       -.00042080
  40.00        -10.00        -151.09             .00        .00021329       -.00000000

      RY( 1)=      248.91
      RY( 2)=      151.09
      RAMZ( 1)=  -1956.22
      C1=-.00000000
      C2= .00000000
```

Test problem 5

This problem consists of a homogeneous circular shaft, three inches in diameter at the center and tapered to one and a half inches in diameter at both ends. The total weight of the shaft is 100# and a uniformly distributed load totaling 200# is placed on the central section. The shaft is supported by a uniformly distributed reaction at each end. Points A and B are points of zero deflection. This problem was approximated for the program as shown in Fig. 12.

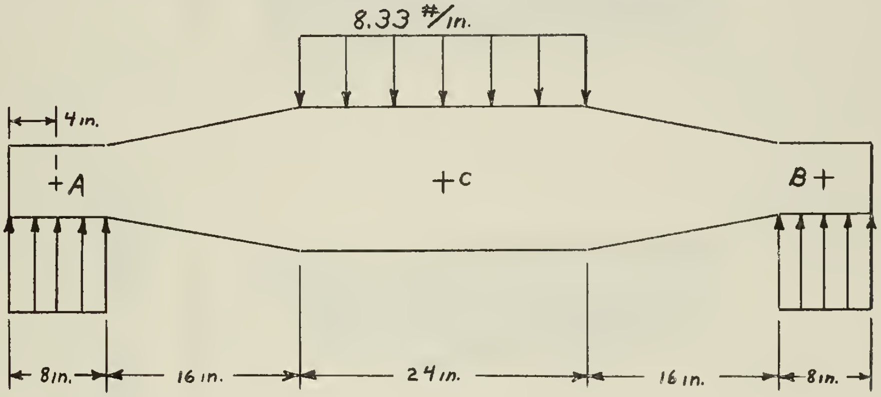


Fig. 12

Weight of shaft:

| | | |
|---|---|---|
| 0" - 8" | -0.5315#/in. | |
| 8" - 24" | -0.5315 to -2.081#/in. | (linear) |
| 24" - 36" | -2.081#/in. | |

Bending compliance $\left(\frac{1}{EI}\right)$:

| | | | | | |
|---|---|---|---|---|---|
| 0" - 8" | $0.1420\text{x}10^{-6}$ | | $1/\#\text{-in.}^2$ | | |
| 8" - 12" | $0.1420\text{x}10^{-6}$ | to | $0.0583\text{x}10^{-6}$ | $1/\#\text{-in.}^2$ | (linear) |
| 12" - 16" | $0.0583\text{x}10^{-6}$ | to | $0.0281\text{x}10^{-6}$ | $1/\#\text{-in.}^2$ | (linear) |
| 16" - 20" | $0.0281\text{x}10^{-6}$ | to | $0.0152\text{x}10^{-6}$ | $1/\#\text{-in.}^2$ | (linear) |
| 20" - 24" | $0.0152\text{x}10^{-6}$ | to | $0.0089\text{x}10^{-6}$ | $1/\#\text{-in.}^2$ | (linear) |
| 24" - 36" | $0.0089\text{x}10^{-6}$ | | $1/\#\text{-in.}^2$ | | |

Shear compliance $\left(\frac{K}{AG}\right) = 0.$

SOLUTION TO TEST PROBLEM 5

```
DISTANCE         LOAD           SHEAR          MOMENT          SLOPE           DEFLECTION

    .00          18.22            .00            .00         -.00144040        .00573400
   4.00          18.22          72.96         145.75         -.00141280       -.00000000
   8.00           -.53         145.83         583.33         -.00121954       -.00534754
  12.00           -.92         142.93        1161.37         -.00088582       -.00953806
  16.00          -1.31         138.48        1724.71         -.00064188       -.01256778
  20.00          -1.69         132.48        2267.15         -.00047103       -.01477502
  24.00         -10.41         124.93        2782.50         -.00035015       -.01640427
  28.00         -10.41          83.29        3198.94         -.00024330       -.01759611
  32.00         -10.41          41.64        3448.80         -.00012461       -.01833491
  36.00         -10.41           -.00        3532.09         -.00000000       -.01858513
  40.00         -10.41         -41.64        3448.80          .00012461       -.01833491
  44.00         -10.41         -83.29        3198.94          .00024330       -.01759611
  48.00          -2.08        -124.93        2782.50          .00035015       -.01640427
  52.00          -1.69        -132.48        2267.15          .00047103       -.01477502
  56.00          -1.31        -138.48        1724.71          .00064188       -.01256778
  60.00           -.92        -142.93        1161.37          .00088582       -.00953806
  64.00          18.22        -145.83         583.33          .00121954       -.00534754
  68.00          18.22         -72.87         145.75          .00141280        .00000000
  72.00          18.22           -.00           -.00          .00144040        .00573400

   RY( 1)=         .08
   RY( 2)=         .08
   C1=-.00144040
   C2= .00573400
```

Test problem 6

This hyperstatic problem consists of a beam with a very unlikely configuration as shown in Fig. 13. The bending compliance varies as shown in Fig. 14. Shear deflection has been omitted, i.e., $\frac{K}{AG} = 0$.

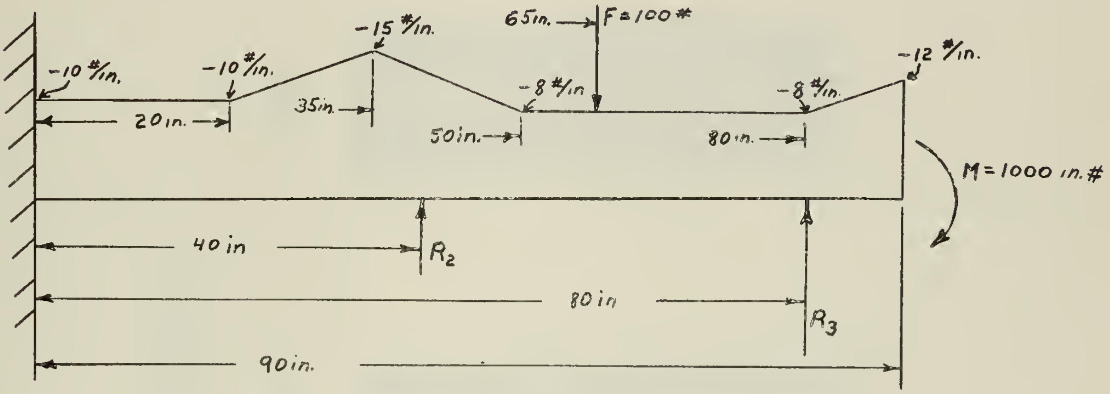


Fig. 13

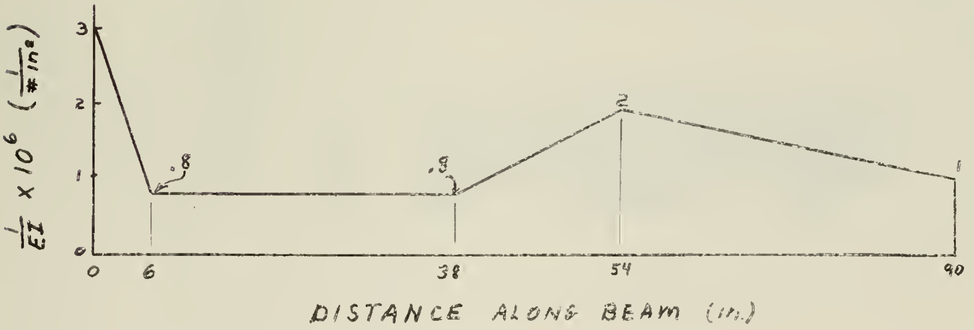


Fig. 14

Test problem 7

This problem is identical to problem number 6 except the beam was turned end for end.

SOLUTION TO TEST PROBLEM 6

```
DISTANCE          LOAD          SHEAR         MOMENT          SLOPE          DEFLECTION

    .00          -10.00        218.61       -1672.36       -.00000000        .00000000
   4.50          -10.00        173.61        -789.88       -.00576422       -.01429420
   9.00          -10.00        128.61        -109.89       -.00804624       -.04663760
  13.50          -10.00         83.61         367.59       -.00716320       -.08194572
  18.00          -10.00         38.61         642.57       -.00377395       -.10733002
  22.50          -10.83         -7.43         714.18        .00112008       -.11363611
  27.00          -12.33        -59.56         565.97        .00612216       -.09705796
  31.50          -13.83       -118.43         168.01        .00927688       -.06127469
  36.00          -14.53       -183.66        -509.94        .00796860       -.02022418
  40.50          -12.43        208.18       -1250.21        .00018866        .00036344
  45.00          -10.33        156.95        -432.21       -.00544019       -.01398517
  49.50           -8.23        115.18         176.53       -.00610080       -.04120624
  54.00           -8.00         79.12         613.58       -.00456710       -.06571866
  58.50           -8.00         43.12         888.61       -.00181456       -.08044870
  63.00           -8.00          7.12        1001.65        .00163650       -.08100193
  67.50           -8.00       -128.88         702.68        .00495289       -.06575553
  72.00           -8.00       -164.88          41.72        .00634141       -.03945104
  76.50           -8.00       -200.88        -781.25        .00505885       -.01268946
  81.00           -8.40         91.80       -1437.40        .00065355        .00124103
  85.50          -10.20         49.95       -1115.42       -.00435623       -.00670383
  90.00          -12.00           .00       -1000.00       -.01445347       -.04653770

     RY( 1)=      218.61
     RY( 2)=      452.51
     RY( 3)=      328.88
     RAMZ( 1)=  -1672.36
     C1= .00000000
     C2= .00000000
```

SOLUTION TO TEST PROBLEM 7

```
DISTANCE        LOAD         SHEAR        MOMENT         SLOPE       DEFLECTION

    .00       -12.00          .00       -1000.00      .01445346     -.04653768
   4.50       -10.20       -49.95       -1115.42      .00435623     -.00670377
   9.00        -8.40       -91.80       -1437.40     -.00065355      .00124103
  13.50        -8.00       200.88        -781.25     -.00505885     -.01268946
  18.00        -8.00       164.88          41.72     -.00634141     -.03945104
  22.50        -8.00       128.88         702.68     -.00495289     -.06575552
  27.00        -8.00        -7.12        1001.65     -.00163650     -.08100193
  31.50        -8.00       -43.12         888.61      .00181456     -.08044870
  36.00        -8.00       -79.12         613.58      .00456710     -.06571865
  40.50        -8.23      -115.18         176.53      .00610080     -.04120624
  45.00       -10.33      -156.95        -432.21      .00544019     -.01398517
  49.50       -12.43      -208.18       -1250.21     -.00018866      .00036344
  54.00       -14.53       183.66        -509.94     -.00796860     -.02022419
  58.50       -13.83       118.43         168.01     -.00927688     -.06127469
  63.00       -12.33        59.56         565.97     -.00612216     -.09705796
  67.50       -10.83         7.43         714.18     -.00112008     -.11363612
  72.00       -10.00       -38.61         642.57      .00377395     -.10733003
  76.50       -10.00       -83.61         367.59      .00716320     -.08194571
  81.00       -10.00      -128.61        -109.89      .00804624     -.04663761
  85.50       -10.00      -173.61        -789.88      .00576422     -.01429420
  90.00       -10.00      -218.61       -1672.36      .00000000      .00000002

     RY( 1)=      328.88
     RY( 2)=      452.51
     RY( 3)=      218.61
     RAMZ( 1)=     1672.36
     C1= .01445346
     C2=-.04653768
```

# APPENDIX E

## Subroutine GAUSS2

This routine is a modified form of the GAUSS2 used by C. B. Bailey /7/ in his program, LINEQN, for solving simultaneous linear equations by Gaussian elimination. Bailey was solving the matrix equation, Ax=B, with a possibility of 50 vectors B for each matrix A, but this program requires only one vector B with each matrix A. Also since the maximum number of equations in BEAM3 is 22, the size of matrix A was reduced from 100x101 to 22x23.

At each step of the elimination, the value of the diagonal element is compared to a defined "zero". If it is smaller than this quantity, the matrix is considered singular and an error output is returned to the calling program. This is the "matrix singular" test used by BEAM3.

# APPENDIX F

## Listing of program and subroutines

```
      PROGRAM BEAM3
      DIMENSION W(1001),V(1001),AM(1001),Y1(1001),Y(1001),XX(1001),
     1 A(25),B(25),QA(25),QB(25),XFO(100),FY(100),XR(10),RY(10),
     2XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25),
     3AA(22,23),XA(22),PDEFT(10),PSLP(10)
     4 ,ITITLE(10),GA(25),GB(25),AG(25),BG(25)
      COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,RAMZ,AE,BE,EA,EB,NOL,
     1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
      READ  70,C
   70 FORMAT (F20.0)
      READ 71,NOP,NOL,NOF,NOR,NOM,NORM,NOI,NOK
   71 FORMAT(8I4)
      IF(NOL) 51,51,50
   50 READ 72,(A(I),B(I),QA(I),QB(I),I=1,NOL)
   72 FORMAT(4F20.0)
   51 IF(NOF)53,53,52
   52 READ 73,( XFO(N),FY(N),N=1,NOF)
   73 FORMAT (2F20.0)
   53 IF(NOR)55,55,54
   54 READ 74,(XR(M),RY(M),PDEFT(M),M=1,NOR)
   74 FORMAT (3F20.0)
   55 IF(NOM)57,57,56
   56 READ 73,(XM(K),AMZ(K),K=1,NOM)
   57 IF(NORM)59,59,58
   58 READ 74,(RXM(KA),RAMZ(KA),PSLP(KA),KA=1,NORM)
   59  IF(NOI)61,61,60
   60 READ 72,(AE(IA),BE(IA),EA(IA),EB(IA),IA=1,NOI)
   61  IF(NOK)1,1,62
   62  READ 72,(AG(IB),BG(IB),GA(IB),GB(IB),IB=1,NOK)
    1 NOR1  = NOR+1
      C1=0.
      C2= 0.
      NORM2 =NORM  + 2
      NORM3 =NORM  + 3
      NOQ  = NOR  +NORM
      NOQ1 = NOQ  + 1
      NOQ2 = NOQ  + 2
```

```
      NOQ3 = NOQ + 3
      EPSX=10.**(-8)
       DO 2 K=1,NOR
  2    AA(1,K) = 1.
       DO 3 K= NOR1,NOQ2
  3    AA(1,K) = 0.
       AAA=0.
      IF(NOF)45,45,35
 35   DO 4 N=1,NOF
  4    AAA= AAA + FY(N)
 45   IF(NOL)48,48,46
 46   DO 5 I=1,NOL
  5    AAA = AAA + ((QA(I)+QB(I))/2.)* (B(I)-A(I))
 48   AA(1,NOQ3)=-AAA
       DO 6 K =1,NOR
  6    AA(2,K) = C-XR(K)
       IF(NORM)9,9,7
  7    DO  8 K=NOR1,NOQ
  8   AA(2,K)=1.
  9    DO 10  K=NOQ1,NOQ2
 10    AA(2,K) = 0.
       CALL MOMENT (C,AAA)
       AA(2,NOQ3) =-AAA
       IF (NORM)17,17,11
 11    DO 16 J=3,NORM2
       DO 13 K=1,NOR
       AA(J,K)=0.
       DO 12 IA=1,NOI
      AAA=EA(IA)*ENTON(RXM(J-2),AE(IA),XR(K),1) - EB(IA)*ENTON(RXM(J-2),
     1BE(IA),XR(K),1) +((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*(ENT1N(RXM(J-2)
     2,AE(IA),XR(K),1)-ENT1N(RXM(J-2),BE(IA),XR(K),1))
 12    AA(J,K) = AA(J,K) + AAA
      IF (NOK) 13,13,900
900   DO 910  IB=1,NOK
      IF(NORM-1)902,901,902
902   IF(J-NORM2)901,904,901
901   X=RXM(J-2) -EPSX
```

```
      AAA = GA(IB)* FTON(  X      ,AG(IB),XR(K),0) - GB(IB)* FTON(X
     1    ,BG(IB),XR(K),0) + ((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))* ( FT1N(
     2 X      ,AG(IB),XR(K),0)- FT1N(  X      ,BG(IB),XR(K),0))
      GO TO 910
  904 X=RXM(J-2) +EPSX
      AAA = GA(IB)* FTON(  X      ,AG(IB),XR(K),0) - GB(IB)* FTON(X
     1    ,BG(IB),XR(K),0) + ((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))* ( FT1N(
     2 X      ,AG(IB),XR(K),0)- FT1N(  X      ,BG(IB),XR(K),0))
  910 AA(J,K) =AA(J,K) -AAA
   13  CONTINUE
      IF(NOQ-NOR1)155,135,135
  135 DO 15 K=NOR1,NOQ
       AA(J,K)= 0.
       DO 14 IA=1,NOI
      AAA= EA(IA)*ENTON(RXM(J-2),AE(IA),RXM(K-NOR),0) -EB(IA)* ENTON
     1 (RXM(J-2),BE(IA),RXM(K-NOR),0) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA))
     2) *(ENT1N(RXM(J-2),AE(IA),RXM(K-NOR),0) - ENT1N(RXM(J-2),BE(IA),
     3 RXM(K-NOR),0))
   14  AA(J,K) = AA(J,K) + AAA
   15  CONTINUE
  155 AA(J,NOQ1)=1.
       AA(J,NOQ2)= 0.
      IF(J-NORM2)156,157,156
  156 X=RXM(J-2) - EPSX
      CALL SLOPE(X,AAA)
      GO TO 16
  157 X=RXM(J-2) + EPSX
      CALL SLOPE(X,AAA)
   16  AA(J,NOQ3) =-AAA + PSLP(J-2)
   17  DO 20 J=NORM3,NOQ2
       DO 18 K=1,NOR
       AA(J,K) =0.
      DO 181  IA=1,NOI
       AAA = EA(IA)* DITON(XR(J-NORM2),AE(IA),XR(K),1) -EB(IA)*DITON
     1 (XR(J-NORM2),BE(IA),XR(K),1) +((EB(IA)-EA(IA))/ (BE(IA)-AE(IA)))
     2 *(DIT1N(XR(J-NORM2),AE(IA),XR(K),1) - DIT1N(XR(J-NORM2),BE(IA),
     3 XR(K),1))
```

```
181 AA(J,K) = AA(J,K) + AAA
    IF (NOK)18,18,920
920 DO 930  IB=1,NOK
    AAA =GA(IB)* ENTON(XR(J-NORM2),AG(IB),XR(K),0) - GB(IB)*ENTON
   1 (XR(J-NORM2),BG(IB),XR(K),0) +((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))
   2 *(ENTIN(XR(J-NORM2),AG(IB) ,XR(K),0) -ENTIN(XR(J-NORM2),BG(IB),
   3 XR(K),0))
930 AA(J,K) =AA(J,K) -AAA
 18 CONTINUE
    IF(NOQ-NOR1)195,185,185
185 DO 19 K=NOR1,NOQ
     AA(J,K) = 0.
     DO 19 IA=1,NOI
     AAA = EA(IA)* DITON(XR(J-NORM2),AE(IA),RXM(K-NOR),0) - EB(IA)*
   1 DITON(XR(J-NORM2),BE(IA),RXM(K-NOR),0) +((EB(IA)-EA(IA)) / (BE(IA
   2 )- AE(IA))) * (DITIN(XR(J-NORM2),AE(IA),RXM(K-NOR),0) - DITIN
   3 (XR(J-NORM2),BE(IA),RXM(K-NOR),0))
 19  AA(J,K) = AA(J,K) + AAA
195 AA(J,NOQ1) =XR(J-NORM2)
     AA(J,NOQ2) = 1.
     CALL DEFLECT (XR(J-NORM2),AAA)
 20  AA(J,NOQ3) =-AAA+ PDEFT(J-NORM2)
     CALL GAUSS2(NOQ2,.00000001,AA,XA,K1)
     GO TO (22,21),K1
 21  WRITE OUTPUT TAPE 4,77
 77  FORMAT (16H MATRIX SINGULAR)
    STOP
 22  DO  23 K=1,NOR
 23  RY(K) = XA(K)
     IF(NORM)26,26,24
 24  DO  25 K=NOR1,NOQ
 25  RAMZ(K-NOR) = XA(K)
 26  C1 = XA(NOQ1)
     C2 = XA(NOQ2)
     X=0.
    NOPE =NOP-1
     FNOP = NOP-1
```

```
      DO  27  J=1,NOPE
       CALL LOAD (X,W(J))
       CALL SHEAR (X,V(J))
       CALL MOMENT (X,AM(J))
      X1 =X+EPSX
      CALL SLOPE(X1,Y1(J))
      Y1(J)=Y1(J) +C1
       CALL DEFLECT (X,Y(J))
      Y(J)=Y(J)+C1*X+C2
       XX(J) = X
   27  X = C/FNOP +X
      XX(NOP)=X
        X=X-3.*EPSX
      CALL LOAD(X,W(NOP))
      CALL SHEAR(X,V(NOP))
      CALL MOMENT(X,AM(NOP))
      CALL SLOPE(X,Y1(NOP))
      Y1(NOP)=Y1(NOP)+C1
      CALL DEFLECT(X,Y(NOP))
      Y(NOP)=Y(NOP)+C1*X +C2
      WRITE OUTPUT TAPE 4,76
   76 FORMAT ( 3X,1HJ,7X,4HLOAD,10X,5HSHEAR,10X,6HMOMENT,8X,5HSLOPE,
     1 9X,10HDEFLECTION,8X,1HX)
      WRITE OUTPUT TAPE 4,78,(J,W(J),V(J),AM(J),Y1(J),Y(J),XX(J),J=1,
     1NOP)
   78 FORMAT(I4,6E15.8)
      IF (NOR)282,282,280
  280 PRINT 281,(M,RY(M),M=1,NOR)
  281 FORMAT ( 3HRY(,I2,2H)=,E20.8)
  282 IF(NORM)288,288,283
  283 PRINT 284,(KA,RAMZ(KA),KA=1,NORM)
  284 FORMAT (5HRAMZ(,I2,2H)=,E20.8)
  288 PRINT 286,C1
  286 FORMAT(5X,3HC1=,E20.8)
      PRINT 287,C2
  287 FORMAT(5X,3HC2=,E20.8)
  285 CONTINUE
```

```
STOP
END
```

```
      SUBROUTINE LOAD(X,W)
      DIMENSION A(25),B(25),QA(25),QB(25),XFO(100),FY(100),XR(10),RY(10)
     1,XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25)
     2,GA(25),GB(25),AG(25),BG(25)
      COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,RAMZ,AE,BE,EA,EB,NOL,
     1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
      EPSX=10.**(-8)
      W=0.
    1 DO 5 I=1,NOL
    2 IF(X+EPSX-B(I))3,3,5
    3 IF(X+EPSX-A(I))5,4,4
    4 WW=QA(I)+((QB(I)-QA(I))*(X-A(I)))/(B(I)-A(I))
      W=W+WW
    5 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SHEAR(X,V)
      DIMENSION A(25),B(25),QA(25),QB(25),XFO(100),FY(100),XR(10),RY(10)
     1,XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25)
     2,GA(25),GB(25),AG(25),BG(25)
      COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,RAMZ,AE,BE,EA,EB,NOL,
     1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
      EPSX=10.**(-8)
      V=0.
      IF(NOL)7,7,1
    1 DO 6 I=1,NOL
      IF(X-A(I))6,2,2
    2 IF(X+EPSX-B(I))4,4,3
    3 VV= QA(I)*(X-A(I) )+((QB(I)-QA(I))*((X-A(I))**2))/(2.*(B(I)-A(I)))
     1-QB(I)*(X-B(I)) -((QB(I)-QA(I))*((X-B(I))**2))/(2.*(B(I)-A(I)))
      GO TO 5
    4 VV=+QA(I)*(X-A(I) )+((QB(I)-QA(I))*((X-A(I))**2))/(2.*(B(I)-A(I)))
    5 V=V+VV
    6 CONTINUE
    7 IF(NOF)10,10,75
   75 DO 9 N=1,NOF
      IF(X+EPSX-XFO(N)) 9,9,8
    8 VV=FY(N)
      V=V+VV
    9 CONTINUE
   10 IF(NOR)13,13,105
  105 DO 12 M=1,NOR
      IF(X+EPSX-XR(M))12,12,11
   11 VV= RY(M)
      V=V+VV
   12 CONTINUE
   13 CONTINUE
       RETURN
      END
```

```
      SUBROUTINE MOMENT(X,AM)
      DIMENSION A(25),B(25),QA(25),QB(25),XFO(100),FY(100),XR(10),RY(10)
     1,XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25)
     2,GA(25),GB(25),AG(25),BG(25)
      COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,RAMZ,AE,BE,EA,EB,NOL,
     1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
      EPSX=10.**(-8)
      AM=0.
      IF(NOL)8,8,2
    2 DO 7  I=1,NOL
      IF(X-A(I))7,3,3
    3 IF(X+EPSX-B(I))5,5,4
    4 AMM=(+QA(I)/2.)*((X-A(I))**2)+((QB(I)-QA(I))*((X-A(I))**3))/(6.*(
     1B(I)-A(I)))-(QB(I)/2.)*((X-B(I))**2)- (QB(I)-QA(I))*((X-B(I))**3)
     2/(6.*(B(I)-A(I)))
      GO TO 6
    5 AMM=(+QA(I)/2.)*((X-A(I))**2)+((QB(I)-QA(I))*((X-A(I))**3))/(6.*
     1(B(I)-A(I)))
    6 AM=AM+AMM
    7 CONTINUE
    8 IF(NOF)12,12,9
    9 DO 11 N=1,NOF
      IF(X+EPSX-XFO(N))11,11,10
   10 AMM= FY(N)*(X-XFO(N))
      AM=AM+AMM
   11 CONTINUE
   12 IF(NOR)16,16,13
   13 DO 15 M=1,NOR
      IF(X+EPSX-XR(M))15,15,14
   14 AMM= RY(M)*(X-XR(M))
      AM=AM+AMM
   15 CONTINUE
   16 IF(NOM)28,28,17
   17 DO 19 K=1,NOM
      IF(X+EPSX-XM(K))19,19,18
   18 AMM=AMZ(K)
      AM=AM+AMM
```

```
19 CONTINUE
28 IF(NORM) 32,32,29
29 DO 31 KA=1,NORM
   IF(X+EPSX-RXM(KA))31,31,30
30 AMM=RAMZ(KA)
   AM=AM+AMM
31 CONTINUE
32 CONTINUE
   RETURN
   END
```

```
   SUBROUTINE SLOPE(X,Y1)
   DIMENSION A(25),B(25),QA(25),QB(25),XFC(100),FY(100),XR(10),RY(10)
  1,XM(5C),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25)
  2,GA(25),GB(25),AG(25),BG(25)
   COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,RAMZ,AE,BE,EA,EB,NOL,
  1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
   Y1=0.
   DO 17  IA=1,NOI
   IF(X-AE(IA))17,17,2
 2 IF(NOL)5,5,3
 3 DO  4  I=1,NOL
20 EYY = EA(IA)*((QA(I)/2.)*ENTON(X,AE(IA),A(I),2) +((QB(I)-QA(I))/
  1(6.*(B(I)-A(I))))*(ENTON(X,AE(IA),A(I),3)-ENTON(X,AE(IA),B(I),3))
  2 -(QB(I)/2.)*ENTON(X,AE(IA),B(I),2))  +((EB(IA)-EA(IA))/(BE(IA)-
  3 AE(IA)))*((QA(I)/2.)*ENT1N(X,AE(IA),A(I),2) +((QB(I)-CA(I)) /
  4(6.*(B(I)-A(I))))*(ENT1N(X,AE(IA),A(I),3)-ENT1N(X,AE(IA),B(I),3))
  5 -(QB(I)/2.)*ENT1N(X,AE(IA),B(I),2))
   Y1=Y1+EYY
21 EYY= -EB(IA)*((QA(I)/2.)*ENTON(X,BE(IA),A(I),2) +((QB(I)-CA(I))/
  1(6.*(B(I)-A(I))))*(ENTON(X,BE(IA),A(I),3)-ENTON(X,BE(IA),B(I),3))
  2 -(QB(I)/2.)*ENTON(X,BE(IA),B(I),2))  -((EB(IA)-EA(IA))/(BE(IA)-
  3 AE(IA)))*((QA(I)/2.)*ENT1N(X,BE(IA),A(I),2) +((QB(I)-QA(I)) /
  4(6.*(B(I)-A(I))))*(ENT1N(X,BE(IA),A(I),3)-ENT1N(X,BE(IA),B(I),3))
  5 -(QB(I)/2.)*ENT1N(X,BE(IA),B(I),2))
   Y1=Y1+EYY
 4 CONTINUE
 5 IF(NOF)8,8,6
 6 DO  7 N=1,NOF
22 EYY= EA(IA)*FY(N)*ENTON(X,AE(IA),XFO(N),1) -EB(IA)*FY(N)*ENTON(X,
  1BE(IA),XFO(N),1) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*FY(N)*
  2(ENT1N(X,AE(IA),XFO(N),1) - ENT1N(X,BE(IA),XFO(N),1))
   Y1=Y1+EYY
 7 CONTINUE
 8 IF(NOR)11,11,9
 9 DO 10 M=1,NOR
23 EYY= EA(IA)*RY(M)*ENTON(X,AE(IA),XR(M),1)-EB(IA)*RY(M)* ENTON(X,
  1BE(IA),XR(M),1) +((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*RY(M) *
```

```
     2(ENT1N(X,AE(IA),XR(M),1)- ENT1N(X,BE(IA),XR(M),1))
      Y1=Y1+EYY
   10 CONTINUE
   11 IF(NOM) 14,14,12
   12 DO 13  K=1,NOM
   24 EYY= EA(IA)*AMZ(K)*ENTON(X,AE(IA),XM(K),0) -EB(IA)*AMZ(K)*ENTON(X,
     1BE(IA),XM(K),0) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))* AMZ(K) *
     2 (ENT1N(X,AE(IA),XM(K),0)- ENT1N(X,BE(IA),XM(K),0))
      Y1=Y1+EYY
   13 CONTINUE
   14 IF(NORM) 17,17,15
   15 DO 16  KA= 1,NORM
   25 EYY=EA(IA)*RAMZ(KA)*ENTON(X,AE(IA),RXM(KA),0)-EB(IA)*RAMZ(KA)*
     1 ENTON(X,BE(IA),RXM(KA),0) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA))) *
     2RAMZ(KA)*(ENT1N(X,AE(IA),RXM(KA),0)- ENT1N(X,BE(IA),RXM(KA),0))
      Y1=Y1+EYY
   16 CONTINUE
   17 CONTINUE
      IF(NOK)595,595,500
  500 DO 59C  IB=1,NOK
      IF (X -AG(IB))590,59C,509
  509 IF (NCL)530,530,510
  510 DO 52C  I=1,NOL
   26 EYY = GA(IB)*(QA(I)*FTON(X,AG(IB),A(I),1) + ((QB(I) -QA(I) ) /
     1 (2.*(B(I)-A(I))))*(FTON(X,AG(IB),A(I),2)-FTON(X,AG(IB),B(I),2))
     2 -QB(I)*FTON(X,AG(IB),B(I),1)) + ((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))
     3 *(QA(I)*FT1N(X,AG(IB),A(I),1) + ((QB(I)-QA(I))/(2.*(B(I)-A(I))))
     4* (FT1N(X,AG(IB),A(I),2) -FT1N(X,AG(IB),B(I),2)) - QB(I)* FT1N
     5 (X,AG(IB),B(I),2))
      Y1 =Y1 -EYY
   27 EYY =-GB(IB)*(QA(I)*FTON(X,BG(IB),A(I),1) + ((QB(I) -CA(I)) /
     1 (2.*(B(I)-A(I))))*( FTON(X,BG(IB),A(I),2)-FTON(X,BG(IB),B(I),2))
     2-QB(I)* FTON(X,BG(IB),B(I),1))- ((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))
     3*(QA(I)* FT1N(X,BG(IB),A(I),1)+ ((QB(I)-QA(I))/(2.*(B(I)-A(I))))
     4*( FT1N(X,BG(IB),A(I),2)- FT1N(X,BG(IB),B(I),2)) - QB(I)*  FT1N
     5 (X,BG(IB),B(I),2))
      Y1 =Y1 -EYY
```

```
520 CONTINUE
530 IF(NOF)560,560,540
540 DO 55C  N=1,NOF
 28 EYY=GA(IB)*FY(N)* FTON(X,AG(IB),XFO(N),0) -GB(IB)*FY(N)* FTON(X,
   1BG(IB),XFO(N),0) + ((GB(IB)-GA(IB)) / (BG(IB)-AG(IB))) *  FY(N)*
   2( FT1N(X,AG(IB),XFO(N),0) -  FT1N(X,BG(IB),XFO(N),0))
    Y1 =Y1 -EYY
550 CONTINUE
560 IF(NOR)590,590,570
570 DO 58C  M=1,NOR
 29 EYY =GA(IB)*RY(M)* FTON(X,AG(IB),XR(M),0) -GB(IB)*RY(M)* FTON(X,
   1BG(IB),XR(M),0) + ((GB(IB)-GA(IB)) / (BG(IB)-AG(IB))) * RY(M) *
   2( FT1N(X,AG(IB),XR(M),0) -  FT1N(X,BG(IB),XR(M),0))
    Y1 =Y1 -EYY
580 CONTINUE
590 CONTINUE
595 CONTINUE
    RETURN
    END
```

```
    SUBROUTINE DEFLECT(X,Y)
    DIMENSION A(25),B(25),QA(25),QB(25),XFO(100),FY(100),XR(10),RY(10)
   1,XM(50),AMZ(50),RXM(10),RAMZ(10),AE(25),BE(25),EA(25),EB(25)
   2,GA(25),GB(25),AG(25),BG(25)
    COMMON A,B,QA,QB,XFO,FY,XR,RY,XM,AMZ,RXM,RAMZ,AE,BE,EA,EB,NOL,
   1 NOR,NOF,NOM,NORM,NOI,NOK,GA,GB,AG,BG
    Y=0.
    DO 17  IA=1,NOI
    IF(X-AE(IA))17,17,2
  2 IF(NOL)5,5,3
  3 DO  4  I=1,NOL
 20 EYY = EA(IA)*((QA(I)/2.)*DITON(X,AE(IA),A(I),2) +((QB(I)-QA(I))/
   1(6.*(B(I)-A(I))))*(DITON(X,AE(IA),A(I),3)-DITON(X,AE(IA),B(I),3))
   2 -(QB(I)/2.)*DITON(X,AE(IA),B(I),2))  +((EB(IA)-EA(IA))/(BE(IA)-
   3 AE(IA)))*((QA(I)/2.)*DIT1N(X,AE(IA),A(I),2) +((QB(I)-QA(I)) /
   4(6.*(B(I)-A(I))))*(DIT1N(X,AE(IA),A(I),3)-DIT1N(X,AE(IA),B(I),3))
   5-(QB(I)/2.)*DIT1N(X,AE(IA),B(I),2))
    Y=Y+EYY
 21 EYY= -EB(IA)*((QA(I)/2.)*DITON(X,BE(IA),A(I),2) +((QB(I)-QA(I))/
   1(6.*(B(I)-A(I))))*(DITON(X,BE(IA),A(I),3)-DITON(X,BE(IA),B(I),3))
   2 -(QB(I)/2.)*DITON(X,BE(IA),B(I),2))  -((EB(IA)-EA(IA))/(BE(IA)-
   3AE(IA)))*((QA(I)/2.)*DIT1N(X,BE(IA),A(I),2) +((QB(I)-QA(I)) /
   4(6.*(B(I)-A(I))))*(DIT1N(X,BE(IA),A(I),3)-DIT1N(X,BE(IA),B(I),3))
   5 -(QB(I)/2.)*DIT1N(X,BE(IA),B(I),2))
    Y=Y+EYY
  4 CONTINUE
  5 IF(NOF)8,8,6
  6 DO  7 N=1,NOF
 22 EYY= EA(IA)*FY(N)*DITON(X,AE(IA),XFO(N),1) -EB(IA)*FY(N)*DITON(X,
   1 BE(IA),XFO(N),1) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*FY(N)*
   2(DIT1N(X,AE(IA),XFO(N),1) - DIT1N(X,BE(IA),XFO(N),1))
    Y=Y+EYY
  7 CONTINUE
  8 IF(NOR)11,11,9
  9 DO 10 M=1,NOR
 23 EYY= EA(IA)*RY(M)*DITON(X,AE(IA),XR(M),1)-EB(IA)*RY(M)* DITON(X,
   1BE(IA),XR(M),1) +((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*RY(M) *
```

```
  2(DIT1N(X,AE(IA),XR(M),1)- DIT1N(X,BE(IA),XR(M),1))
   Y=Y+EYY
 10 CONTINUE
 11 IF(NOM)14,14,12
 12 DO 13  K=1,NOM
 24 EYY=EA(IA)*AMZ(K)*DITON(X,AE(IA),XM(K),0) -EB(IA)*AMZ(K)*DITON(X,
  1BE(IA),XM(K),0) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA)))*AMZ(K)*
  2 (DIT1N(X,AE(IA),XM(K),0)- DIT1N(X,BE(IA),XM(K),0))
   Y=Y+EYY
 13 CONTINUE
 14 IF(NORM) 17,17,15
 15 DO 16  KA= 1,NORM
 25 EYY=EA(IA)*RAMZ(KA)*DITON(X,AE(IA),RXM(KA),0)-EB(IA)*RAMZ(KA)*
  1 DITON(X,BE(IA),RXM(KA),0) + ((EB(IA)-EA(IA))/(BE(IA)-AE(IA))) *
  2RAMZ(KA)*(DIT1N(X,AE(IA),RXM(KA),0)- DIT1N(X,BE(IA),RXM(KA),C))
   Y=Y+EYY
 16 CONTINUE
 17 CONTINUE
   IF(NOK)595,595,500
500 DO 59C  IB=1,NOK
   IF (X -AG(IB))590,590,509
509 IF (NCL)530,530,510
510 DO 52C  I=1,NCL
 26 EYY=GA(IB)*(QA(I)* ENTON(X,AG(IB),A(I),1) + ((QB(I) -CA(I) ) /
  1 (2.*(B(I)-A(I))))*(ENTON(X,AG(IB),A(I),2)-ENTON(X,AG(IB),B(I),2))
  2-QB(I)*ENTON(X,AG(IB),B(I),1)) + ((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))
  3*(QA(I)*ENT1N(X,AG(IB),A(I),1) + ((QB(I)-CA(I))/(2.*(B(I)-A(I))))
  4*(ENT1N(X,AG(IB),A(I),2)-ENT1N(X,AG(IB),B(I),2)) - QB(I)*ENT1N
  5 (X,AG(IB),B(I),2))
   Y=Y-EYY
 27 EYY=-GB(IB)*(QA(I)*ENTON(X,BG(IB),A(I),1) +((QB(I)-QA(I))/
  1(2.*(B(I)-A(I))))*(ENTON(X,BG(IB),A(I),2)-ENTON(X,BG(IB),B(I),2))
  2-QB(I)*ENTON(X,BG(IB),B(I),1))- ((GB(IB)-GA(IB))/(BG(IB)-AG(IB)))
  3*(QA(I)*ENT1N(X,BG(IB),A(I),1)+ ((QB(I)-QA(I))/(2.*(B(I)-A(I))))
  4*(ENT1N(X,BG(IB),A(I),2)-ENT1N(X,BG(IB),B(I),2)) - QB(I)* ENT1N
  5 (X,BG(IB),B(I),2))
   Y=Y-EYY
```

```
520 CONTINUE
530 IF(NOF)560,560,540
540 DO 550  N=1;NOF
 28 EYY=GA(IB)*FY(N)*ENTON(X,AG(IB),XFO(N),0) -GB(IB)*FY(N)*ENTON(X,
   1BG(IB),XFO(N),0) + ((GB(IB)-GA(IB)) / (BG(IB)-AG(IB))) *  FY(N)*
   2(ENT1N(X,AG(IB),XFO(N),0) - ENT1N(X,BG(IB),XFO(N),0))
    Y=Y-EYY
550 CONTINUE
560 IF(NOR)590,590,570
570 DO 580  M=1,NOR
 29 EYY =GA(IB)*RY(M)*ENTON(X,AG(IB),XR(M),0) -GB(IB)*RY(M)*ENTON(X,
   1BG(IB),XR(M),0) + ((GB(IB)-GA(IB)) / (BG(IB)-AG(IB))) * RY(M) *
   2(ENT1N(X,AG(IB),XR(M),0) - ENT1N(X,BG(IB),XR(M),0))
    Y=Y-EYY
580 CONTINUE
590 CONTINUE
595 CONTINUE
    RETURN
    END
```

```
  FUNCTION FTON(X,A,B,N)
  FTON = 0.
  IF(X-A)3,3,1
1 IF(X-B)3,3,2
2 FTON=(X-B)**N
3 RETURN
  END
```

```
  FUNCTION FT1N (X,A,B,N)
  FT1N =0.
  IF (X-A)3,3,1
1 IF (X-B)3,3,2
2 FT1N = (X-B)**(N+1) + (B-A)*((X-B)**N)
3 RETURN
  END
```

```
     FUNCTION  ENTON (X,A,B,N)
     FN=N
     ENTON = 0.
    IF(X-A)5,5,1
  1 IF(X-B)5,5,2
  2  IF(A-B)3,3,4
  3  ENTON = (X-B)**(N+1)/(FN+1.)
     GO TO 5
  4  ENTON = (X-B)**(N+1)/(FN+1.) - (A-B)**(N+1)/(FN+1.)
  5  RETURN
    END
```

```
     FUNCTION  ENT1N(X,A,B,N)
     FN=N
     ENT1N = 0.
    IF(X-A)5,5,1
  1 IF(X-B)5,5,2
  2  IF(A-B)3,3,4
  3  ENT1N = (X-B)**(N+2)/(FN+2.) +  (B-A)*(X-B)**(N+1)/(FN+1.)
     GO TO 5
  4  ENT1N = (X-B)**(N+2)/(FN+2.) + (B-A)*(X-B)**(N+1)/(FN+1.)
   1 -(A-B)**(N+2)/(FN+2.) -((B-A)*(A-B)**(N+1))/(FN+1.)
  5  RETURN
     END
```

```
      FUNCTION DITON(X,A,B,N)
      FN=N
      DITON =0.
      IF(X-A)5,5,1
    1 IF(X-B)5,5,2
    2 IF(A-B)3,3,4
    3 DITON = (X-B)**(N+2)/((FN+1.)*(FN+2.))
      GO TO 5
    4 DITON = (X-B)**(N+2)/((FN+1.)*(FN+2.)) -(X*(A-B)**(N+1))/(FN+1.)
     1-(A-B)**(N+2)/((FN+1.)*(FN+2.))  + (A*(A-B)**(N+1))/(FN+1.)
    5 RETURN
      END
```

```
      FUNCTION  DIT1N(X,A,B,N)
      FN=N
      DIT1N=0.
      IF(X-A)5,5,1
    1 IF(X-B)5,5,2
    2 IF(A-B)3,3,4
    3 DIT1N =(X-B)**(N+3)/((FN+2.)*(FN+3.)) + ((B-A)*(X-B)**(N+2))/
     1((FN+1.)*(FN+2.))
      GO TO 5
    4 DIT1N = (X-B)**(N+3)/((FN+2.)*(FN+3.)) + ((B-A)*(X-B)**(N+2))/
     1((FN+1.)*(FN+2.))-(X*(A-B)**(N+2))/(FN+2.)-(X*(B-A)*(A-B)**(N+1))/
     2 (FN+1.) - (A-B)**(N+3)/((FN+2.)*(FN+3.)) - (B-A)*(A-B)**(N+2)/
     3 ((FN+1.)*(FN+2.)) + (A*(A-B)**(N+2))/(FN+2.) + (A*(B-A)*(A-B)**
     4 (N+1))/(FN+1.)
    5 RETURN
      END
```

```
         SUBROUTINE  GAUSS2(N,EP,A,X,KER)
     DIMENSION A(22,23) ,X(22)
      NPM=N+1
  10  DO 34 L=1,N
      KP=0
      Z=0.C
      DO 12 K=L,N
      IF(Z-ABSF(A(K,L)))11,12,12
  11  Z=ABSF(A(K,L))
      KP=K
  12  CONTINUE
      IF(L-KP)13,20,20
  13  DO 14 J=L,NPM
      Z=A(L,J)
      A(L,J)=A(KP,J)
  14  A(KP,J)=Z
  20  IF(ABSF(A(L,L))-EP)50,50,30
  30  IF(L-N)31,40,40
  31  LP1=L+1
      DO 34 K=LP1,N
      IF(A(K,L))32,34,32
  32  RATIO=A(K,L)/A(L,L)
      DO 33 J=LP1,NPM
  33  A(K,J)=A(K,J)-RATIO*A(L,J)
  34 CONTINUE
  40  DO 43 I=1,N
      II=N+1-I
      DO 43 J=1,1
      JPN=J+N
      S=0.0
      IF(II-N)41,43,43
  41  IIP1=II+1
      DO 42 K=IIP1,N
  42  S=S+A(II,K)*X(K)
  43  X(II)  =(A(II,JPN)-S)/A(II,II)
      KER=1
75   RETURN
```

```
50 KER=2
   RETURN.
   END
```

[illegible]
[illegible]
[illegible]